# BOOK OF ABSTRACTS

# 5th INTERNATIONAL CONFERENCE ON COMPUTATIONAL MATHEMATICS AND ENGINEERING SCIENCES

08-10 JUNE
2021 Van, TURKEY

# THE FIFTH INTERNATIONAL CONFERENCE ON COMPUTATIONAL MATHEMATICS AND ENGINEERING SCIENCES (CMES-2021), VAN/TURKEY, JUNE 08-10, 2021

The **Fifth International Conference on Computational Mathematics and Engineering Sciences (CMES-2021)** will be held in Van Yüzüncüyil University from **June 08-10, 2021 in Van, Turkey**. It provides an ideal academic platform for researchers and professionals to discuss recent developments in both theoretical, applied mathematics and engineering sciences. This event also aims to initiate interactions among researchers in the field of computational mathematics and their applications in science and engineering, to present reccent developments in these areas, and to share the computational experiences of our invited speakers and participants.

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# MESSAGE FROM THE GENERAL CHAIRS


Dear Conference Attendees,

We would like to welcome you to the **5th International Conference on Computational Mathematics and Engineering Sciences (CMES-2021)** at Van Yüzüncüyıl University from June 08-10, 2021 in Van, Turkey. This year, the conference includes 300 extended abstracts, out of 300 submissions received in response to the call for papers, selected by the Program Committee. The program features keynote talks by distinguished speakers such as Abdon Atangana from Free State University, South Africa, Dumitru Baleanu, from Institute of Space Sciences, Magurele-Bucharest, Romania, Carlo Cattani from Tuscia University, Viterbo Italy, Juan Luis García Guirao from Technical University of Cartagena, Spain. Oscar Castillo from Tijuana Institute of Technology, Tijuana, Mexico, Ali Yousef from Kuwait College of Science and Technology, Kuwait, Vatan Karakaya from Yildiz Technical University, Istanbul, Turkey, Ali Rostami Photonics and Nanocrystals Research Lab (PNRL), University of Tabriz, Tabriz, Iran. The conference also comprises contributed sessions, posters sessions and research highlights.

We would like to thank the Program Committee members and external reviewers for volunteering their time to review and discuss submitted abstracts. We would like to extend special thanks to the Honorary, Scientific and Organizing Committees for their efforts in making CMES-2021 a successful event. We would like to thank all the authors for presenting their research studies during our conference. We hope that you will find CMES-2021 interesting and intellectually stimulating, and that you will enjoy meeting and interacting with researchers around the world.

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Applied Mathematics,
Financial Mathematics,
Control Theory,
Game Theory
Modeling of Bio-systems
for Optimization and Control,
Linear and Nonlinear
programming and Dynamics,
Artificial Intelligence,
Geometry and Its Applications,
Analysis and Its Applications,
Statistics and Its Applications,
Mathematics Education and Its Applications,
Algebra and Its Applications.
Engineering Sciences
Computer Science

Information technology
Electrical and Electronic Engineering
Ordinary, Partial, Stochastic and Delay
Differential Equations
Chaos and Dynamical Systems
Numerical methods and
scientific programming
Fractional Calculus and Applications,
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Topology and Its Application
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# PLENARY & INVITED SPEAKERS TALKS

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## FRACTIONAL DIFFERENTIATION AND INTEGRATION ABOVE POWER LAW SOME NEW DEVELOPMENTS

**Abstract**

To capture more complexities in nature some new differential and integral operators were suggested very recently. These differential operators are defined as fractal derivative of order beta of a convolution of power law, exponential decay and the generalized Mittag-Leffler function. I will represent some new theoretical results and their applications to capture nature.

***Keywords:*** Generalized Mittag-Leffler function

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# SOME NEW TRENDS IN FRACTIONAL CALCULUS

## Abstract

Fractional calculus is an emerging field of mathematics with important applications in many branches of science and engineering. In this talk some basic problems of the fractional calculus will be reviewed and some new trends will be presented. Illustrative examples from fractional mathematical biology will be given.

***Keywords:*** **Fractional calculus, fractional mathematical biology.**

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# A REVIEW ON WAVELET FRACTIONAL CALCULUS

## Abstract

In this talk a review on wavelets and their application in fractional calculus, will be discussed. The main properties of the most popular wavelet families will be given, by taking into account of their characteristic features in the Fourier domain. It will be shown that the fractional operators based on wavelets have a very simple expression thus opening new frontiers in the solution of fractional differential problems.

***Keywords:*** **Wavelets, local fractional derivative, wavelet series**

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# SOME NEW TRENDS IN FRACTIONAL CALCULUS

## Abstract

The aim of the present contribution provide a generalization of the classical Shannon-Whittaker-Katel'niko's theorem for a class of non band-limited signals which plays a central role in the signal theory, the Gaussian map is the unique function which reachs the minimum of the product of the temporal and frecuential width. This solves a conjecture stated by Boas in 1972.

***Keywords:*** **Shannon–Whittaker–Kotel'nikov's Theorem; recomposi- tion of chemical products, signal theory.**

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# OPTIMIZATION OF TYPE-2 FUZZY SYSTEMS: THEORY AND APPLICATIONS

## Abstract

Type-2 fuzzy systems are powerful intelligent models based on the theory of fuzzy sets, originally proposed by Prof. Zadeh. Most real-world applications up to now are based on type-1 fuzzy systems, which are built based on the original (type-1) fuzzy sets that extend the concept of classical sets. Type-2 fuzzy sets extend type-1 fuzzy sets by allowing the membership to be fuzzy, in this way allowing a higher level of uncertainty management. Even with the current successful applications of type-1 fuzzy systems, now several papers have shown that type-2 is able to outperform type-1 in control, pattern recognition, manufacturing and other areas. The key challenge in dealing with type-2 fuzzy models is that their design has a higher level of complexity, and in this regard the use of bio-inspired optimization techniques is of great help in finding the optimal structure and parameters of the type-2 fuzzy systems for particular applications, like in control, robotics, manufacturing and others. Methodologies for designing type-2 fuzzy systems using bio-inspired optimization in different areas of application are presented as illustration. In particular, we will cover Bee Colony Optimization, Particle Swarm Optimization, Gravitational Search and similar approaches to the optimization of fuzzy systems in control applications, robotics and pattern recognition [1, 2, 3, 4, 5]. Finally, we will also consider using fuzzy logic for enhancing the performance of metaheuristics, where also good results have been achieved.

***Keywords:*** **Type-2 Fuzzy Logic, Optimization, Fuzzy Logic, Metaheuristics**

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# ASYMPTOTIC CHARACTERISTICS OF THE THREE-STAGE PROCEDURE WHEN THE NORMAL COEFFICIENT OF VARIATION IS KNOWN WITH SIMULATION

## Abstract

This paper studies the three-stage[Hall, Ann Stat 9(6):1229–1238, 1981]sequential procedure's asymptotic characteristics when the normal population coefficient of variation is known. We estimate the population mean of the normal distribution using the Searls estimator [Searls, J. Amer Statist Ass 59, 308: 1225-1226, 1964]. We tackle three estimation problems; first, minimum risk point estimation using a squared-error loss function plus linear sampling cost, where we find the asymptotic risk and regret. Second, we find the asymptotic fixed-width confidence interval for the mean, and third, we discuss the sensitivity of the procedure to detect any potential shift that occurs in the population mean by finding the asymptotic operating characteristic function. The theoretical results reveal that the asymptotic characteristics of the three-stage procedure depend on the numerical value of the coefficient of variation. A Monte Carlo simulation is conducted using Microsoft Developer Studio software to study the procedure's performance using different values of the coefficient of variation. The simulation results agree with the theoretical findings.

***Keywords:*** **Confidence interval, loss function, minimum risk point estimation, operating characteristic function, risk, Searls estimator, a three-stage procedure**

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# THE FORMATION STAGES OF MATHEMATICAL MODELING IN TERMS OF MATH PHILOSOPHY

## Abstract

The perception of mathematical objects by the human mind has been one of the fundamental questions of philosophy in general and of philosophy of mathematics in particular throughout history. Many schools of thought have put forward different opinions on how the concepts of number and geometric, which are abstaract entities, are perceived by the human mind in the form of "counting" and "measuring". In the philosophical trend led by Descartes, the mathematical verification and model resource was Analytic geometry, which is formed by the combination of algebra and geometry. In this trend, mathematical verification has turned into a verification method in the form of Clarity, Analysis, Synthesis, and Counting. Kant was also one of the philosophers who opened a detailed discussion on how mathematical concepts are formed. Describing mathematical objects as synthetic a priori, Kant claimed that the information obtained by man through the five senses in the body, world turns into perception and then becomes an abstract entity through perception. He argued that both numbers and geometric objects transform from the object to the abstract being and from the abstract being into the objective rules with the relationship of the visible and conceivable realm. However, Frege, one of Kant's followers, defended the thesis that, unlike Kant, numbers are a logical inference, therefore should be analytical a priori, not synthetic a priori. Frege defended the thesis that numbers can be constructed by purely logical rules without a visual space and that "the object of the mind is the mind." Under this view, he argued that the geometric concepts of mathematics require a sensory perception, on the other hand numbers can be deduced by the rules of logic without sensory perception, and this constructed system can represent the relations of real life. In terms of mathematical philosophy, numbers and geometric shapes, which are mathematical objects, have been described as entities designed in the human mind, dependent or independent on human sense perceptions, but have a representation in the material world with a model. This phenomenon will be tried to be explained through the abstract representation of the triangle, which consists only of points as an abstract entity, and its models as representing the triangle in the visible world.

***Keywords:*** **Noumenon, Phenomenon, Counting, Measuring**

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# MATHEMATICAL MODELING OF HIGH-EFFICIENCY SOLAR CELLS USING SELECTIVE ENERGY CONTACTS

## Abstract

One of the most important and challenging loss mechanisms of solid-state photovoltaics is the heat production of energetic carriers excited by high-energy incident photons. This presentation shows that if carriers are extracted at their high energies before cooling down due to scattering, the conversion efficiency can be noticeably enhanced and therefore the generated heat will be down considerably. To increase the efficiency of a single-band gap solar cell in this work, selective energy contacts are introduced to a p-i-n structure to extract hot carriers (high energy carriers). A selective energy contact solar cell is made up of many collecting contacts with particular energy differences from the conduction band of the cell. In other words, each contact could extract carriers with a special range of energies. The concept of selective energy contact solar cells is to collect high energy carriers, i.e. electrons in this case, within a range of energies onto external electrodes before they cool down. The comparison between conventional solar cells and selective energy contact solar cells shows a significant enhancement in electron collection and efficiency. Based on simulation results, it is observed that the efficiency of the selective energy contact solar cell has been enhanced substantially exceeding almost twice as much as a conventional solar cell's and reaching a significant 34% efficiency.

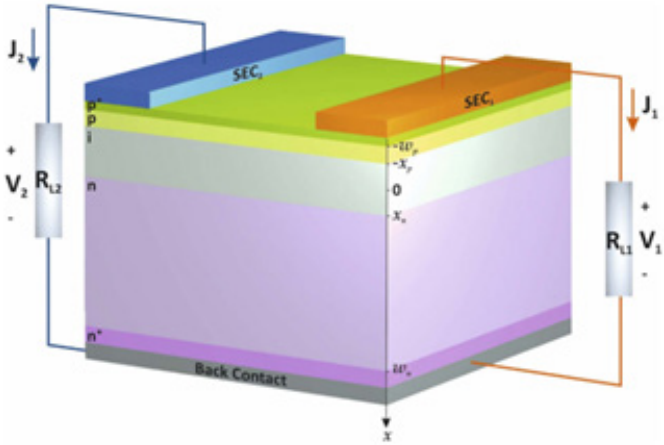


Fig. 1. Schematics of the high-efficiency solar cell

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# On the Solution of a Parabolic Equation Using the Ritz Method

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**Abstract:** In this study, Ritz method has been applied for approximate solution of a parabolic equation. The Ritz method was analyzed for error analysis and the solutions were compared on the graph. It is explained on the table that the change in error depends on the number of basic system functions used in the Ritz method.

**Keywords:** Ritz method, parabolic equation, error analysis

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# A FINITE DIFFERENCE ANALYSIS OF BOUNDARY VALUE PROBLEMS FOR SINGULARLY PERTURBED EQUATIONS ON BAKHVALOV MESH

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**Abstract**

In this paper we present a special uniform finite difference method on an B-mesh (Bakhvalov type mesh) to solve the boundary value problem for singularly perturbed second order differential equations. The solution of foresold problem, exhibits the boundary layer on the left and right-hand side of the domain due to the presence of singular perturbation parameter $\varepsilon$. For this problem finite difference scheme on a special non-uniform mesh, whose solution converges point-wise independently of the $\varepsilon$ small parameter is constructed and analyzed. The stability and convergence analysis of the method are investigated. The scheme is uniformly convergent, i.e., their convergence is independent of the small perturbation parameter. An error analysis on the scheme shows that the method is of second order convergent in the discrete maximum norm independent of the perturbation parameter, i.e., the scheme are uniformly convergent. Several numerical examples are also given to demonstrate the efficiency of B-mesh to validate the theoretical aspects.

***Keywords:*** Difference scheme; Bakhvalov mesh; Singular perturbation; Uniform convergence.

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# NEW INTEGRAL INEQUALITIES FOR QUASI-CONVEX AND P-FUNCTIONS VIA ATANGANA-BALEANU FRACTIONAL INTEGRAL OPERATORS

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**Abstract**

To prove new integral inequalities, we need to integral identities created by different methods and operators. In this article, we have used an integral identity that was given with the help of Atangana-Baleanu integral operators and proved various integral inequalities for quasi-convex functions and P-functions.

***Keywords:*** Convex function, Hölder inequality, Young inequality, power mean inequality, Atangana-Baleanu fractional integral operators

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# COMPARISON OF NUMERICAL METHODS FOR SOLVING STOCHASTIC DIFFERENTIAL EQUATIONS WITH TIME DELAY

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**Abstract**

In this paper, we have presented computational methods for solving stochastic differential equations (SDDEs) with distuributed delay term in the drift and diffusion cofficient. Our purpose is comparatively investigates some numerical methods to solve stochastic delay differential equations. We indicate the nature of the methods of interest and examine convergence of them. By presenting some numerical experiments we illustrate the theorical results and finally the results numerical methods are supported with graphs and error tables and discussed about which method is useful and superior and in which cases these methods can be used.

***Keywords:*** Stochastic delay differential equations; Strong solutions; Numerical methods; Strong and weak convergence.

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# Efficient Image Encryption Scheme Based on 4-Dimensional Chaotic Maps

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**Abstract**

In this work, we propose a novel image encryption scheme based on 4-dimensional chaotic maps. This scheme is built up from a robust light shuffling phase and a masking phase. Both phases operate on image-blocks and they are independent form each other. The scheme uses measures of central tendency to mix image blocks to enhance robustness against cryptanalytic attacks. While the encryption phase is highly sensitive to its secret key and plain-image, the decryption phase is robust against noise and cropping of the cipher-image. Simulations demonstrate the high performance of the proposed scheme and its high security level against well-known cryptanalytic attacks. Furthermore, they show its superiority over existing schemes.

***Keywords**:* Cryptography; Chaos; Cat map; Pseudorandom numbers; Image encryption.

**The full paper has been recently published in Informatica ISSN print 0868-4952.**

*Kanso, Ali, Mohammad Ghebleh, and Abdullah Alazemi. "Efficient Image Encryption Scheme Based on 4-Dimensional Chaotic Maps." Informatica 31, no. 4 (2020): 793-820.*

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Please refer to the above paper.

# Exact Boundary Controllability of Nodal profile for a Hyperbolic Problem

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**Abstract:** In this paper, we give some results about the exact boundary controllability of nodal profile, originally proposed in [4], for a hyperbolic system with the constant coefficient. Firstly, we introduce the dual version of the controllability problem, which leads to an observability inequality for the corresponding adjoint system. We prove a sufficient condition in order to have a control function by applying duality approach.

**Keywords:** Wave Equation, Exact Boundary Controllability, Nodal Profile Control.

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# Optimal Control of the Initial Condition in a Heat Conductivity Problem

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**Abstract:** This study deals with obtaining a solution for a control problem of the initial condition in a parabolic system. It is demonstrated that the optimal solution for the considered optimal control problem is exists and unique. After obtaining the gradient of the cost functional utilizing from the adjoint problem, the necessary condition for the optimal solution is derived.

**Keywords:** Optimal Control, Heat Equation, Frechet Differentiability.

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# The Global Attractivity of Linearizable Scalar Difference Equations

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**Abstract**

In this paper, I will consider the linearized difference equation

$$\vec{x}_{n+1} = \sum_{i=1-l}^{k} g_i \vec{x}_{n-i}$$

where $k \in \{1,2,\ldots\}$ $and$ $l \in \{1,2,\ldots\}$ $and$ $g_i$ in general, depend on $n$ and the state variables $\vec{x}_k$ . I use method of linearization to extend some of the results about the global attractivity and asymptotic stability of scalar equation to the case of vector equation. The results contain the global attractivity in the delicate cases when the sum of the norms of $g_i$ is less than 1 and the sum of the scalar functions $g_i$ is equal to 1. I will also illustrate the results with many examples that include possible models in population dynamics, as all transition functions are Holling type functions for related results.

***Keywords:*** *attractivity, difference equations, discrete dynamical system, global, linear fractional, rational, stability*

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# Investigation of the Wave Solutions of the Nonlinear Partial Differential Equations with the Modified Exponential Function Method

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**Abstract**

In this study, the wave solutions related to the Dullin-Gottwold-Holm equation were obtained using the modified exponential function method. Solution functions obtained as a result of calculations include hyperbolic, trigonometric, and rational functions. Two-dimensional, three-dimensional, contour graph and density graphs representing the characteristic feature of the solution functions obtained by determining the appropriate parameters are drawn.

***Keywords:*** The wave solutions; the Dullin-Gottwold-Holm equation; the modified exponential function method.

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# On the Mixture of Garima and Lomax Distributions

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In this paper, some basic statistical properties of the mixture garima lomax distribution have been obtained. Garima distribution is mainly used to explain the new lifetime distribution. It can be used in a wide variety of fields, including biology, engineering and medicine. The Lomax or Pareto II (the shifted Pareto) distribution was pioneered to model business failure data by Lomax. This distribution has found wide application in a variety of fields such as income and wealth inequality, size of cities, actuarial science, medical and biological sciences, engineering, lifetime and reliability modeling. As a result, the statistical properties obtained are presented with graphics.

**Key Words:** Garima-Lomax Distribution, Order Statistics, Moment Generating Function and Characteristic Function

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# VARIOUS CHARACTERIZATIONS FOR QUATERNIONIC MANNHEIM CURVES IN THREE-DIMENSIONAL EUCLIDEAN SPACE

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**Abstract**

In this study, quaternionic curves in 3-dimensional Euclidean space have examined. Firstly, algebraic properties of quaternions and their basic definitions and theorems are given. Later, some characterizations of the quaternionic Mannheim curves in the 3-dimensional Euclidean space have obtained.

***Keywords:*** Real quaternion, Spatial quaternion, Quaternion algebra, Mannheim partner curves

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# TESTING THE RELIABILITY OF A MACHINE WITH NON-HOMOGENEOUS POISSON PROCESS

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**Abstract**

The deterioration processes of independent and identically distributed repairable systems are often modeled by the Non-Homogeneous Poisson Process. The basic model in this process is the Power Law model. In this study, an application has been made to test the reliability of a bank's ATM. Parameter estimation was made using the maximum likelihood method in order to determine the model used when the data are not independent and identically distributed that is, in accordance with the Non-Homogeneous Poisson Process. Again in this study, using the obtained parameters, the expected number of failures, the next downtime and the reliability values for various time intervals are calculated.

As a result, it is concluded that the Non-Homogeneous Poisson Process is a suitable and sufficient modeling in time-dependent analysis for its use as a model in repairable systems and for deterioration data.

***Keywords:*** Maximum Likelihood Method, Power Law Model, Reliability.

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# SOME APPLICATIONS OF GREEN'S FUNCTION ON TIME SCALES

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**Abstract**

In this study, we deal with Green's function for second order dynamic boundary value problems on time scales. Green's function is established on time scale in many different situations. Here, it will be constructed in accordance with our problem and the discussed problems will be solved. These problems and their obtained results show how important Green's function is in mathematical physics on time scales (see [1-7]).

***Keywords:*** Time Scales, Green's Function

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# A Generalization of Randic Incidence and Laplacian Incidence Energies

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**Abstract.** For a connected graph $G$ and a real number $\alpha \neq 0,1$, the graph invariants $s_\alpha$ and $\sigma_\alpha$ are defined as the sum of $\alpha th$ powers of normalized Laplacian eigenvalues and the sum of $\alpha th$ powers of normalized signless Laplacian eigenvalues, respectively. In this study, we generalize some bounds for $s_\alpha, \sigma_\alpha$, Randic incidence energy $I_R E$ and Laplacian incidence energy $LIE$ of graphs (connected non-bipartite, connected bipartite).

**Keywords:** Energy, Randic Incidence Energy, Laplacian Incidence Energy

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# ε- Uniform Numerical Method for The Singularly Perturbed Nonlinear Mixed Type Integro-Differential Equations

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**Abstract**

This article deals with the singularly perturbed nonlinear mixed integro-differential equations. The features of the exact solution are analyzed. Difference scheme which is accomplished by the method of integral identities with using of the interpolating quadrature rules with weight functions and remainder term in integral form is presented for both grade mesh and the piecewise equidistant mesh. The error analysis is succesfully carried out in the discrete maximum norm, independently of the perturbation parameter. The academical results are implemented on two examples and the obtained outcomes are compared for both B-type and S-type meshes.

***Keywords:*** Bakhvalov Mesh, Error Estimate, Difference Scheme, Mixed Integro-Differential Equation, Singular Perturbation, Shishkin Mesh.

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# UNIFORMLY NUMERICAL METHOD FOR SINGULARLY PERTURBED SOBOLEV PROBLEM WITH THIRD TYPE BOUNDARY CONDITION ON BAKHVALOV MESH

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**Abstract**

In this study, we consider the following linear singularly perturbed initial-third value Sobolev problem:

$$Lu = L_1\left[\frac{\partial^2 u}{\partial t^2}\right] + L_2 u = f(x,t), \qquad (x,t) \in D = (0,L)\times(0,t]$$

$$(x,0) = \varphi(x), \qquad \frac{\partial u}{\partial t}(x,0) = \psi(x), \qquad x\epsilon[0,l]$$

$$L_* u = \mu(t)u(0,t) - \sqrt{\varepsilon}\frac{\partial u}{\partial x}(0,t) = g(t), \qquad u(l,t) = 0$$

where

$$L_1\left[\frac{\partial^2 u}{\partial t^2}\right] = -\varepsilon\frac{\partial^4 u}{\partial t^2 \partial x^2} + a(x)\frac{\partial^2 u}{\partial t^2}, \qquad L_2 u = -\varepsilon\frac{\partial^4 u}{\partial x^2} + b(x,t)u$$

and $0 < \varepsilon < 1$ is a small parameter; the functions $a, b, f, \varphi, \psi, \mu$ and $g$ are sufficiently smooth and $a(x) >\propto> 0$, $\mu(t) > \mu_* > 0$. For this problem, it is shown that the solution is depend on initial datas of problem according to $L_2$ and maximum norm. We construct a classical tree-level scheme on an adaptive mesh which was accomplished by the method of integral identities with using of basis functions and interpolating quadrature rules with weight and remainder term in integral form were presented. The convergence and error estimates for a finite difference scheme in an adaptive mesh are obtained.

***Keywords:*** Adaptive mesh, Boundary layers, Difference scheme, Singular perturbation, Sobolev problem.

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# NUMERICAL METHOD FOR SINGULARLY PERTURBED CONVECTION-DIFFUSION PERIODIC BOUNDARY VALUE PROBLEM ON SHISHKIN MESH

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**Abstract**

This paper is concerned with following singularly perturbed convection-diffusion problem with periodic boundary conditions:

$$Lu = \varepsilon u'' + a(x)u' + b(x)u = f(x), \quad 0 < x < l \tag{1.1}$$

$$u(0) = u(l), \tag{1.2}$$

$$u'(l) - u'(0) = \frac{A}{\varepsilon} \tag{1.3}$$

where $\varepsilon$ is a small positive parameter, the coefficients $a(x) \geq \alpha > 0$, $b(x) \geq \beta > 0$, $f(x)$ are sufficiently smooth functions and $A$ is a given constant. The solution $u$ generally has boundary layer near $x = 0$ and $x = l$.

The aim of this study is to present finite difference method for (1.1)-(1.3) problem. In this context, asymptotic estimations are obtained to discrete maximum norm. Using interpolating quadrature rules and basis functions, classical finite difference scheme is established on a Shishkin mesh. The error analysis of the difference scheme is achieved. Some numerical experiments confirm the theoretical results.

***Keywords:*** Boundary layers, Convection-diffusion problem, Finite difference method, Singular perturbation, Shishkin mesh.

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# Grand Canonical Monte Carlo Modeling of Bio-MOFs for Anesthetic Xe Recovery Applications

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## Abstract

Xenon (Xe), an noble gas, has been successfully used as an anesthetic agent in the clinical industry. Xe can be, indeed, an ideal anesthetic due to its high stability, low solubility in blood, very low interactions with drug molecules, and minimal side effects. One of the main factors hindering the widespread use of Xe in anesthesia is its relatively high market price (Banerjee et al., 2018) resulting from its energy-requiring purification, e.g. cryogenic distillation of air. One of the suggested methods to reduce the operating cost and improve the use of Xe in clinical applications is to recover Xe from the exhaled gas mixture and feed it back into the inhalation gas mixture during anesthesia operation (Stanley, 2000). The direction that drives our work is to investigate the separation performance of Xe from exhale gas mixtures ( $CO_2$, $N_2$, $O_2$) using metal-organic frameworks as adsorbents. We apply Grand Canonic Monte Carlo (GCMC) Simulations for modeling 20 MOFs that have biological organic linkers, e.g. Bio-MOFs, and various transition metals, e.g., Zn, Ni, Co. First, the charges of 20 BioMOFs are calculated using the Density Functional Theory and Density Derived Electrostatic and Chemical (DDEC) charge method in order to accurately capture electrostatic interactions between Bio-MOF atoms and gas molecules. Then, GCMC simulations at pressures ranging from 0.1 to 10 bar and at 298 K are applied for modeling adsorption isotherms of each gas and Xe adsorption selectivities for each Bio-MOF. We achieve high Xe adsorption and high Xe adsorption selectivity for a range of BioMOFs under consideration. For instance, LUSHOX01 possesses Xe adsorption selectivities as 309.326, 1.0820, and 17.783 for $Xe/N_2$, $Xe/CO_2$, and $Xe/O_2$ binary gas mixtures, respectively. According to the limited study available in the literature, we obtain relatively large Xe adsorption loading and Xe adsorption selectivity for all binary gas mixtures of Xe. Therefore, BioMOFs can be considered as promising adsorbent materials for Xe separation from exhale gas mixtures motivating the use of Xe in the clinical industry.

***Keywords:*** Density Derived Electrostatic and Chemical (DDEC) charge method, metal organic frameworks, gas separation

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# Analysis of Fractional KGZ Equations Using a Different Method

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**Abstract**

In this study, the solution of the equation, Klein-Gordon-Zakharov equations, which is called the q-homotopy analysis transformation method (q-HATM) and defines the interaction between Langmuir waves in plasma and ion-acoustic waves, is given.

The solution was achieved by combining the q-homotopy analysis technique with the Laplace transform. To observe the efficiency and applicability of the solution, this unified nonlinear system was analyzed in different fractional order. Numerical simulation was performed for different fractional lines with changes in variables to show the accuracy and reliability of the method under investigation. The results show that the envisaged plan is highly methodical, very effective, easy to implement and accurate.

***Keywords:*** Laplace transform; Caputo derivative; Klein-Gordon-Zakharov equations; q-Homotopy analysis method.

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# Mechanical and phonon properties of half-Heusler ZrIrSi alloy from density functional theory

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**Abstract**

The structural, elastic and phonon properties of ZrIrSi half-Heusler alloy has been investigated using the generalized gradient approximation method within density functional theory. The ground-state properties, including, lattice constant and bulk modulus are in good agreement with the available data. The elastic constants $C_{ij}$ are computed using the stress–strain technique. According to the results for elastic constants, this alloy meets the Born mechanical stability criteria. Also, according to the Pugh criteria, it was found to have a ductile structure and to show anisotropic behaviour. The phonon dispersion relations of ZrIrSi alloy are calculated for the first time using the density functional theory and the linear response method. The half-Heusler ZrIrSi alloy is found to be stable, according to the phonon calculation. Furthermore, internal free energy, entropy, specific heat capacity at constant volume and vibrational free energy changes of ZrIrSi alloy were analysed and discussed between the temperature range of 0–800 K using the quasi-harmonic approximation.

***Keywords:*** Ab-initio; DFT; elastic constant; phonon; thermodynamic.

# CURVES ACCORDİNG TO ORTHOGONAL ALTERNATIVE FRAME

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**Abstract**

Curves have a very important in differential geometry studies. It is well known that one of the important problems of a regular curve in Euclidean 3-space is the characterization of smooth curves. Our firstly aim in this study is to define an orthogonal alternative frame different from the Frenet frame. Also we obtain various characterizations for some special curves according to orthogonal alternative frame such as helix, slant helix, C-slant helix, rectifying curve, curve pairs.

***Keywords:*** Orthogonal frame, Helix, Slant Helix, Rectifying curve, Curve pairs

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# ROSEN CONTINUED FRACTIONS AND ELEMENTS OF EXTENDED HECKE GROUPS

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**Abstract**

Hecke groups $H(\lambda)$ are infinite class of subgroups of $PSL(2,\mathbb{R})$ and generated by two linear fractional transformations;

$$T(z) = -\frac{1}{z} \text{ and } U(z) = z + \lambda$$

In [1] Erich Hecke proved that $H(\lambda)$ is discrete if and only if $\lambda \geq 2$ or $\lambda_q = 2\cos\frac{\pi}{q}$ where $q \geq 3$ integer. Elements of Hecke groups are orientation preserving isometries of the upper half of the complex plane. The extended Hecke group $\overline{H}(\lambda)$ is the group obtained by adding the reflection element$R(z) = \frac{1}{\bar{z}}$ to the generators of $H(\lambda)$.

There are impressive relations between continued fractions and Hecke groups. In [2] Rosen introduced $\lambda$-continued fractions for $\lambda \epsilon \mathbb{R}$. Owing to this viewpoint Rosen revealed a criteria for the membership problem for $H(\lambda)$.

In this talk we describe types of elements in extended Hecke groups. After we give some fixed point results for extended Hecke groups and relations with Rosen $\lambda$-continued fractions. We obtain parabolic and reflection elements as a word in generators that fixes a given real number.

***Keywords:*** Extended Hecke Groups, Continued Fractions, Fixed Points

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# MULTI-OBJECTIVE FUZZY-STOCHASTIC MATHEMATICAL PROGRAMMING APPROACH FOR FLEXIBLE JOB SHOP SCHEDULING PROBLEM WITH SEQUENCE DEPENDENT SETUP TIMES

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**Abstract**

This study addresses a multi-objective Flexible Job Shop Scheduling Problem with Sequence-Dependent Setup Times (SDST-FJSSP) under a mixed fuzzy and stochastic types of uncertainties. Uncertainty forms of randomness and fuzziness are handled simultaneously to deal with the stochastic due dates, fuzzy processing and setup times. In fact, the proposed mathematical model is an extended version of the original model that was first developed by Mousakhani (2013) for SDST-FJSSP. In this research, the proposed model aims to minimize the total tardiness and maximum completion time concurrently. To do this, a non-preemptive goal programming approach is applied as a multi-objective optimization technique. Additionally, probabilistic uncertainty is considered as a chance-constrained stochastic program and the possibilistic uncertainty is handled by a fuzzy mathematical programming approach. In fact, fuzzy processing and setup times are determined as triangular fuzzy numbers (TFNs). The proposed fuzzy-stochastic MIP model is converted to its crisp equivalent form by well-known transformation approaches in the literature. Finally, in order to show validity and practicality of the proposed fuzzy-stochastic MIP model, an illustrative example is presented and its optimization results via LINGO 18 are also reported.

***Keywords:*** Flexible job-shop scheduling, sequence dependent setup times and fuzzy-stochastic mixed-integer programming.

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# FUZZY-STOCHASTIC MATHEMATICAL PROGRAMMING APPROACH FOR AN INTEGRATED LOT-SIZING AND SUPPLIER SELECTION PROBLEM WITH QUANTITY DISCOUNTS AND BACKORDERING OPTIONS

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**Abstract**

This study addresses an integrated lot-sizing and supplier selection problem with multiple items, time periods, quantity discounts and backordering options under uncertain environments. The main purpose of this study is to cope with the mixed fuzzy and stochastic uncertainties embedded in demands, budget limits and storage capacities. Because, these data cannot be determined precisely in most of the real-life applications. Based on this motivation, a fuzzy-stochastic mixed-integer linear programming (FSMIP) model is developed in order to handle stochastic demands, fuzzy budget limits and storage capacities. The original model of the stated problem was formerly proposed by Alfares & Turnadi (2018) based on a case study of a petrochemical company that operated in a process industry. The proposed fuzzy-stochastic model aims to minimize total cost which consists of the ordering, purchasing, transportation, inventory holding and shortage costs. By making use of the well-known transformation approaches in the literature, the proposed fuzzy-stochastic MIP model is converted to its crisp equivalent form and then illustrated by a hypothetical example.

***Keywords:*** Integrated lot-sizing & supplier selection problems, all-units quantity discount, shortages, backordering and fuzzy-stochastic mixed-integer programming.

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# ON THE RECURSIVE SEQUENCE

$$x_{n+1}=\frac{x_{n-29}}{1+x_{n-4}x_{n-9}x_{n-14}x_{n-19}x_{n-24}}$$

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**Abstract**

The behaivour of the solutions of the following system of difference equations is examined,

$$x_{n+1}=\frac{x_{n-29}}{1+x_{n-4}x_{n-9}x_{n-14}x_{n-19}x_{n-24}},$$

where the initial conditions are positive real numbers.

***Keywords:*** Difference equation, recursive sequence, rational difference equation.

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# ON THE RECURSIVE SEQUENCE

$$x_{n+1}=\frac{x_{n-23}}{1+x_{n-3}x_{n-7}x_{n-11}x_{n-15}x_{n-19}}$$

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**Abstract**

The behaivour of the solutions of the following system of difference equations is examined,

$$x_{n+1}=\frac{x_{n-23}}{1+x_{n-3}x_{n-7}x_{n-11}x_{n-15}x_{n-19}},$$

where the initial conditions are positive real numbers. Moreover, we gave a numerical example of to the solution the related difference equation.

***Keywords:*** Difference equation, recursive sequence, rational difference equation.

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# An Overview of Kinematics in Lorentz Space

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**Abstract**

In this paper, we show that the constraint manifolds of spacelike and timelike mechanisms on the Lorentz plane and hyperbolic sphere $H_0^2$ are L-ellipsoid and L-hyperboloid, respectively. Besides, we find that the constraint manifolds of spacelike (resp. timelike) mechanisms on the Lorentz sphere $S_1^2$ correspond to the constraint manifolds of timelike (resp. spacelike) mechanisms on the sphere $H_0^2$. In the last section, we obtain that the constraint manifolds of spatial mechanisms are the dualizing form of the equations obtained for the hyperbolic sphere.

***Keywords:*** Constraint manifolds, Structure equations, Lorentz sphere, Hyperbolic sphere.

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# THE IMPACT OF GAME-BASED ACTIVITIES ON MATHEMATICAL ACHIEVEMENTS AND PERMENANT LEARNING OF STUDENTS

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## Abstract

Mathematics instruction with activities could be considered as the basic structure of learning process. Because students are expected to be mentally and physically active and conduct tasks in activity-based learning. Thus, it was reported that the development of activities that are consistent with the objectives and their implementation in the classroom environment were important in learning-teaching processes (Özgen and Alkan, 2014). The present study aims to investigate the effects of activities developed to reinforce the learned concepts on student achievements in mathematics and learning permanence. The study was conducted with the semi-experimental design, a quantitative research method. The study sample included 40 (20 in the experimental group and 20 in the control group) students who were attending eighth grade a public school in Diyarbakır province, Turkey during the 2018-2019 academic year. In the mathematics course, the regular curriculum and the "Crossing The Street Game," developed by Bell (1993) were implemented. The activity aims to reinforce the student knowledge on decimal numbers and integer multiplication and dividing topics. The activity could be conducted individually or by two teams. The same rules are as follows: Each player selects two numbers out of 7 numbers presented in the combs. The player should multiply or divide the two numbers using a calculator. If the result is located in one of the combs, the player would place her or his cards on this comb. Thus, the first player or team that would “cross the street” using their connected combs would win the game. The control group was instructed using the program specified in the curriculum. The study data were collected with the achievement test developed by the author. The study data were analyzed with SPSS (Statistical Package for the Social Sciences) 24.0 software and the Kuder-Richardson-20 (KR-20) reliability coefficient of the test was determined as 0.795. Data analysis was conducted with descriptive statistics, dependent and independent groups t-tests. The study findings demonstrated that the activities developed to reinforce the learned concepts had a positive impact on student achievements and ensured learning permanence.

# THE STRENGTH AND IMPORTANCE OF NUMBERS AND NAMES AND THEİR INFLUENCE ON OUR LİFE

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## Abstract

This research sheds light on an important aspect of how numerical science affects human names, and character at every point in human life that the expectations at every stage of life involve a change from person to person such as life expectancy, education, and real individual income, etc. These expectations have been created as a result of life and living needs. From picking exact character names, doing the chores of everyday life, preferring colors and types of clothes, performing religion that has embraced, everything in our life has a tremendous impact on people's choices. Accordingly, are there no other factors affecting our personality? Has it no connection with any concealed or revealed energy that appears to affect humanity? Hence, How does the name affect the person? If so, what is it? What kind of communication? In this paper, we are going to try to make some progress with settling the questions with evidence how the influences of the names on human beings and their connection that have not been fully undiscovered yet, plus what are the names? What are their meanings, and how are they created? What is the numerical equivalent of letters? In the books, Al Futuhat Al Makkiyya (Ibn Arabi) and (Pythagoras) talked about the power of letters and their effect on names accompanied by numbers. Therefore, we are going to establish the existence of its strength, the importance of numbers and names providing evidence, how numbers and letters change behavior, and how it affects a human character. In general, those scientists have also provided information on many topics related to letters and numbers, even today, which all the materials gave references covering research.

***Keywords:*** Abced Account, Number Effect, Name Effect. Our Life

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# SOME PROPERTIES OF A STURM-LIOUVILLE PROBLEM

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**Abstract**

This paper is on the some properties of a Sturm-Liouville problem. The eigenvalues and eigenfunctions of the problem are examined. Some basic properties of the eigenvalues and eigenfunctions are given. Integral equations are found.

***Keywords:*** Sturm-Liouville Theory; Boundary Value Problem; Eigenvalue; Eigenfunction

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# ON FIXED POINT OF SOME GENERALIZED MULTIVALUED MAPPING UNDER FUNCTION SEQUENCES

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**Abstract**

The aim of this paper is to introduce a new fixed point approach on multivalued transformations. As is known, the Banach Contraction principle guarantees the existence of the fixed point of a transformation. Nadler has guaranteed the existence of fixed point by generalizing this idea to multivalued transformations. However, similar methods are not applied for nonexpansive transformations of multivalued transforms. In this study, our aim is to investigate the existence of fixed points of nonexpansive multivalued transformations with the help of shifting distance functions. In addition, it is to give a generalization of this idea using the function sequences. Finally, the study is supported by an interesting example to explain the hypotheses in the work.

***Keywords:*** Fixed point, multivalued contraction mappings, shifting distance function, function sequences

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# NOVEL NUMERICAL APPROACH OF SINGULARLY PERTURBED PROBLEM

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**Abstract**

In this paper, we consider singularly perturbed problem with two integral boundary conditions. This problem numerically is solved by finite difference method on both Bakhvalov and Shishkin meshs. The convergence analysis of the presented method is given in discrete maximum norm. In addition, sample applications proving the accuracy and effectiveness of the proposed method are given.

***Keywords:*** Singular perturbed problem, integral boundary condition, finite difference method, Bakvalov and Shishkin mesh, uniform convergence.

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# A CHEBYSHEV COLLOCATION MATRIX METHOD FOR SOLVING BRATU-TYPE EQUATIONS

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**Abstract**

In this work, we give a collocation matrix method for solving the Bratu-type equation with Chebshev polynomials. This method gives us a chance to reduce such a problem into a nonlinear algebraic equation with the unknown coefficients. Several numerical tests are presented to confirme the accuracy and fruitful of the proposed method. Since our method turned the Bratu equation into a form of matrix-vector equation, it reduces the size of computational work.

***Keywords*:** Bratu-type equation, numerical solution, nonlinear equation, collocation matrix method, shifted Chebyshev polynomials, approximation method

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# BERNSTEIN SERIES APPROACH FOR NONLINEAR FINANCE SYSTEM WITH INPUT TIME DELAY

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**Abstract**

In this study, we have introduced the Bernstein marix-collocation method to find approximate solutions of nonliear finance system with input time delay. The method involves the basic matrix structures gained from the expansion of the series. To demonstrate its advantage on the application, a numerical example is given. In addition, an error estimation technique relating to residual function is performed. All numerical results are presented in the figures and tables.

***Keywords:*** Finance system, approximate solution, Bernstein polynomials and series, matrix-collocation method, residual error analysis

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# INVERSE PROBLEM FOR THE GRAPH INVARIANTS

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**Abstract**

Graph invariant, namely topological index, based on degrees or distances in graphs is the number related to graph. Inverse problem is about for which positive integers there is a graph with these graph invariants of that values. In this work, we consider inverse problem for co-PI index and give a note for Sombor index.

***Keywords:*** *co-PI index, Graph Theory, Degree, Distance, Inverse problem, Sombor indices.*

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# A new Covid 19 multi-strain epidemiological model : analytical study and multi agent based simulations

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**Abstract**

This paper investigates a new multi-strain Covid-19 epidemiological model. We assume that both strains are spreading simulaniously in the population. Each strain has its private parameters : contact rate, recovery rate and attenuation factors. We prove that the disease spread is governed by an ordinary differential equation. Equilibrium points and the basic reproductive number are determined. The paper provides an analytical description of the disease spread's asymptotic behavior and investigates different possible epidemiological scenarios under different hypothesis on each strain parameters. The studied model is simulated using Net-Logo multi-agent program, and the simulations obtained are presented for specific values of the parameters and illustrate analytical results.

***Keywords:*** Multi-strain epidemic models - Ordinary differential equations - Stability analysis - Multi agents approach – Basic reproduction number – Covid 19 modeling.

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# ON HERMITE-HADAMARD TYPE INEQUALITIES FOR $(g,h)$ −PREINVEX DOMINATED FUNCTIONS

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**Abstract**

In this article, we have defined a new class as $(g,h)$ −preinvex dominated function. Then using the definition of $(g,h)$ − preinvex dominated function, we obained $(g,s)$ −preinvex dominated function in the secod sense, $\big(g,Q(K)\big)$ −preinvex dominated function and $\big(g,P(K)\big)$ −preinvex dominated function. Then, we have obtained new Hermite-Hadamard type inequalities for the kinds of preinvex dominated functions just described.

***Keywords:*** Convex-dominated; h-preinvex; Hermite-Hadamard inequality; Preinvex-dominated.

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# HERMITE-HADAMARD TYPE INTEGRAL INEQUALITIES FOR $g$ −PREINVEX DOMINATED FUNCTIONS and $(g, r)$ − PREINVEX DOMINATED FUNCTIONS

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**Abstract**

In this article, we have defined two new class of function by using the definitions of dominated convex functions and preinvex functions. We obtained integral inequalities of Hermite-Hadamard type for newly defined preinvex dominated functions.

***Keywords:*** Preinvex-dominated function; Hermite-Hadamard inequality; Invex; Preinvex; r-preinvex.

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# EXPRESSION OF THE LAW AS A MATHEMATICAL

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**Abstract**

Reaching an ideal order is the common goal of law and mathematics. However, the methods they use to achieve this goal are different. It is an indisputable fact that mathematics is a universal language. Therefore, legal laws can be expressed mathematically by making use of the universality of mathematics. Thus, developments in law find a place for themselves in the wider international arena, thanks to a common language. It is also thought that both the mathematical analysis of the events and the mathematical expression of the laws will contribute to the effective decision-making and deterrent sanctions of the judges in the British American Legal System and the Continental European Legal System. Even though it is thought that way, it is difficult to find the mathematical equivalent of every legal concept.

**Keywords:** Mathematical Statement of Law, Law and Mathematics, Mathematical Modeling, Law and Mathematics in Achieving Ideal Order, Universal Language in Law.

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# GAUSS QUADRATURE RULE OF INTEGRATION

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**Abstract**

In this talk, the Gauss quadrature rule of integration based on the classical orthogonal polynomials (the Jakobi, Laguerre and the Hermite) and its error term will be derived in a unified manner. Integration points which are the roots of classical orthogonal polynomials and the weights of the Gaussian quadrature will be computed with the help of so-called Golub-Welsch algorithm. The method will be applied to a wide range of integrals in order to discuss both the advantages and the disadvantages.

***Keywords:*** Numerical integration, classical orthogonal polynomials, Gaussian quadrature

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# On the Numerical Solution of the Time Fractional Newell-Whitehead-Segel Equations

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**Abstract**

In this study, the time-fractional Newell-Whitehead-Segel (NWS) equation is investigated. Standard and non-standard finite-difference schemes were used to solve the time-fractional Newell-Whitehead-Segel (NWS) equation for cases of $p = 2,3,4$ . The numerical results achieved from proposed methods are compared with exact solutions for different values of fractional order $\alpha$.

***Keywords:*** Newell-Whitehead-Segel Equation (NWS), Standart Finite-Difference Method (SFDM), Non-Standard Finite-Difference Method (NSFDM) .

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# The Solution of First Kind Linear Volterra Integral Equations with $\mathcal{ZZ}$ Transform

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**Abstract:** In this paper, we apply $\mathcal{ZZ}$ transform to solve First Kind Linear Volterra Integral equations. The several examples solve by $\mathcal{ZZ}$ Transform. This means that $\mathcal{ZZ}$ transform is a powerful tool for solving First Kind Linear Volterra Integral equations. In this paper, $\mathcal{ZZ}$ transform for the solution of First Kind Linear Volterra Integral equations presented and in application section of this paper, some applications are given to demostrate the effectiveness of proposed scheme.

**Keywords:** Integral Equations, First Kind Linear Volterra Integral Equation, $\mathcal{ZZ}$ Transform, Convolution Theorem, Inverse $\mathcal{ZZ}$ Transform.

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# On the Inverse problem for singular Sturm-Liouville Operator

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In this work, we give the solution of the inverse problem on two spectras for singular Sturm-Liouville operator. In particulary, we obtain theorem concerning the structure of the difference potentials. Before formulating the main result of this work, we must mention that the analogous inverse problems were examined in the work [1]

We shall consider Sturm -Liouville equation

$$L_1 y = -y'' + q_1(x)y = \lambda y, \quad (0 \le x \le \pi) \qquad (1)$$

with the boudary conditions

$$y(0) = 0, \quad y'(\pi, \lambda) + H_1 y(\pi, \lambda) = 0 \qquad (2)$$

where the function $q_1(x)$ continous and real-valued. Let $\{\lambda_n, \alpha_n\}$ be spectral characteristic of $L_1$ with conditions $(2)$.

Consider a new operator

$$L_2 y = -y'' + q_2(x) = \mu y \qquad (3)$$

Let $\{\mu_n, \beta_n\}$ be spectral characteristic of $L_1$ with conditions $(2)$. Under the assumption, it follows that

$$max|q_1(x) - q_2(x)| \le C.A$$

where $A = \sum_n |\alpha_n \mu_n - \beta_n \lambda_n|$ and $C$ is constant.

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# On $q$-starlike and $q$-convex functions defined by using the function $f_\delta$

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**Abstract**

In this paper we obtain the bounds for certain coefficient functional problems of $q$ −starlike and $q$ −convex functions defined by using the function $f_\delta$ and the concept of $q$-calculus. Also, we derive similar type of inequalities related to lemniscate of Bernoulli.

***Keywords:*** Analytic functions, subordination, $q$-starlike functions, $q$-convex functions, Fekete-Szegö inequality, lemniscate of Bernoulli

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# Extractions of deeper properties of the Conformable Gross–Pitaevskii Equation via two powerful approaches

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**Abstract**

This paper focus on the extractions of travelling wave solutions of the Conformable Gross–Pitaevskii equation via two powerful approaches, namely, sine-Gordon expansion method and modified exponential function method. Many new analytical solutions to the governing model are reported. Under the suitable chosen of parameters of solutions, various simulations are also plotted with the help of computational program.

***Keywords***: Conformable Gross–Pitaevskii equation; sine-Gordon expansion method; Modified exponential function method; Travelling wave solution; Complex function solutions.

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# A Novel Numerical Scheme on Bakhvalov-Shishkin Mesh for Non-linear Singularly Perturbed Volterra Integro-Differential Equations

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**Abstract**

In this work, we focus on a first order non-linear singularly perturbed Volterra integro-differential equation (SPVIDE). We propose a numerical scheme to address non linear SPVIDE type problem. A finite difference scheme is constructed for the problem on Bakhalov-Shishkin mesh. Method of integral identities with exponential basis and interpolating quadrature rules are utilized for the derivation of the proposed scheme. We establish the error estimates of the approximate solution and present that the scheme is O(N^-1) uniformly convergent, where N is the mesh parameter. A quasilinearization technique is applied to difference scheme to deal with nonlinearity of the problem. We also test the scheme on couple of problems and we validate that the scheme is robust and first order.

***Keywords:*** Singularly perturbed; VIDE; Difference schemes; Uniform convergence; Error estimates, Bakhalov-Shishkin mesh, Nonlinearity.

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# ON SOME GEOMETRIC PROPERTIES OF A SEQUENCE SPACE DEFINED AS THE FIELD OF A TRIANGLE MATRIX

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**Abstract**

In this study, for a sequence space defined by a special triangular matrix, I give some matrix transformations between this space with classical sequence spaces. Further, it is gived some geometric properties of this space.

**Keywords:** Paranormed sequence space, domain of a triangle matrix, Banach-Sacks property.

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# Approximate Numerical solution of nonlinear fractional Klein-Gordon equation with Caputo-Fabrizio fractional operator

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**Abstract:** In this present work the fractional homotopy perturbation transform method (FHPTM) on linear and nonlinear fractional Klein-Gordon equation is applied via Caputo-Fabrizio derivative. This method was used directly to solve the fractional models without being liberalization and discretization. The obtained results confirm that the proposed method is valid and reliable for approximate analytic treatment of a wide classes of nonlinear phenomena. In addition, for various orders, the essence of the results obtained is presented. The solution obtained shows that, to analyze the behavior of nonlinear models in science and technology, the projected algorithm is very powerful and simple to apply.

**Keywords:** fractional Klein-Gordon equation, Caputo-Fabrizio derivative, Laplace transform, approximate Numerical solution

***2010 AMS Subject Classification:*** 44A99, 35Q99.

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# On the fluctuations in the System of the Viscosity-Capillarity Regularization

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**Abstract**

In this thesis, some new complex solutions to the system of the SVCR by using two analytical methods. For better understanding of physical meanings of these analytical solutions obtained, figures are plotted byusing package program. Moreover, strain conditions for validity of these complex solutions are presented. In the last section of this paper, a conclusion part is given about the solutions obtained in here.

***Keywords****:* SVCR, Analytical methods; Exponential function solution, Complex function solution, Contour surfaces.

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# New dark, bright and dark-bright solutions to the resonant Davey-Stewartson equations

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**Abstract**

In this article, the sine-Gordon equation expansion method is used to find the analytical traveling wave solution of the (2+1)-dimensional resonant Davey-Stewartson equations system, which is a natural (2+1)-dimensional version of the resonant nonlinear Schröndinger equation. The imaginary (2+1)-dimensional resonant Davey-Stewatson system is converted into a system of nonlinear ordinary differential equations after it the sine-Gordon method is applied, and the homogeneous balance between the order and the highest power of nonlinear terms of the ordinary differential equation is authorized. Finally, the outcomes equations are solved to achieve some new analytical solutions. For different cases as well as for different values of constants, the Wolfram Mathematica software is used to find novel solutions to the resulting system of the nonlinear differential equation. All new solutions of this study are plotted in 2D and 3D. All gained solutions verify the resonant Davey-Stewartson equations.

***Keywords:*** The sine-Gordon method, Davey-Stewatson system, Novel soliton solutions.

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# HIGHER ORDER LINEAR MULTISTEP METHODS FOR FRACTIONAL INITIAL VALUE PROBLEMS WITH IMPROVED STABILITY

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**Abstract**

We consider the numerical approximation of the fractional initial value problem (FIVP)

$$D^{\gamma}y(t) = f(t, y(t)),\ \ t \geq 0,\ \gamma > 0; \qquad y^{(k)}(0) = y_0^{(k)},\ k = 0,1,\cdots, n = [\gamma] + 1,$$

where the fractional derivative is in the sense of Caputo, by fractional linear multistep methods (FLMMs) using the Grunwald approximation with generating function $W_1(z) = (1-z)^{\gamma}$. We derive higher order approximation schemes of order 2 and 4 using the superconvergence property [1] of the Grunwald approximation. Stability properties of the schemes are analysed and show that the stability regions of the proposed schemes are larger than that of the existing FLMM methods[2—4] of same orders.

We also present numberical results to demonstrate the higher orders of the schemes and to verify that the theoretical results obtained are indeed satisfied.

***Keywords:*** Fractional IVP, FLMM, Grunwald Approximation, Stability regions

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# LINEAR MULTISTEP METHODS FOR FRACTIONAL INITIAL VALUE PROBLEMS WITH IMPROVED STABILITY

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**Abstract**

We consider the numerical approximation of the fractional initial value problem (FIVP)

$$D^{\gamma}y(t) = f(t, y(t)),\ \ t \geq 0,\ 1 < \gamma < 2; \qquad y^{(k)}(0) = y_0^{(k)},\ k = 0,12,3,$$

where the fractional derivative is in the sense of Caputo, by fractional linear multistep methods (FLMM). We derive second order implicit and explicit approximation schemes using the recently developped second order shifted Grunwald approximation [2,3,4] for fractional derivatives. Stability properties of the schemes are analysed and shown that the stability regions of the proposed schemes are larger than that of the previously existing FLMM methods of same order[1].

Numerical results are presented to demonstrate the established second order of the schemes and to verify the theoretical results.

***Keywords:*** Fractional IVP, FLMM, Grunwald Approximation, Stability regions

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1. L. Galeone, R. Garrappa, Explicit methods for farctional differential equations and their stability properties, J. comput. Appl. Math. Vol: 228, No:2, 548—560, 2009.
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# WEIGHTED APPROXIMATION FOR KANTOROVICH TYPE $q$-BALAZS-SZABADOS OPERATORS

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**Abstract**

Balázs and Szabados studied approximation properties of the Bernstein type rational functions called the Balázs-Szabados operators. Different $q$-analogues of Balázs-Szabados operators are recently studied by several authors. New Kantorovich type $q$-analogue of the Balázs-Szabados operators are defined by Hamal and Sabancigil as follows:

$$R^*_{n,q}(f,x)=\sum_{k=0}^{n} r_{n,k}(q,x)\int_0^1 f\left(\frac{[k]_q+q^k t}{b_n}\right)d_q t,\ \text{where}\ f:[0,\infty)\to\mathbb{R},\ q\in(0,1),\ a_n=[n]_q^{\beta-1},$$

$$b_n=[n]_q^{\beta},\ 0<\beta\le\frac{2}{3},\ n\in\mathbb{N},\ x\ge 0,\ r_{n,k}(q,x)=\frac{1}{(1+a_n x)^n}\begin{bmatrix} n \\ k \end{bmatrix}_q (a_n x)^k \prod_{s=0}^{n-k-1}\left(1+(1-q)[s]_q\, a_n x\right)$$

Let $B_2[0,\infty)=\{f:[0,\infty)\to R:|f(x)|\le M_f(1+x^2)\}$, where $M_f$ is a constant depending on $f$, $C_2[0,\infty)=B_2[0,\infty)\cap C[0,\infty)$ and $C_2^*[0,\infty)=\left\{f\in C_2[0,\infty):\lim_{x\to\infty}\frac{f(x)}{1+x^2}<\infty\right\}$.

The norm on the space $C_2^*[0,\infty)$ is shown as $\|f(x)\|_2=\sup_{x\in[0,\infty)}\frac{f(x)}{1+x^2}$.

**Theorem 1** Assume that $q=q_n$ satisfies $0<q_n<1$ and $q_n\to 1$ as $n\to\infty$. Then for each $f\in C_2^*[0,\infty)$ we have $\lim_{n\to\infty}\left\|R^*_{n,q_n}(f,x)-f(x)\right\|_2=0$.

**Theorem 2** Let $0<q_n<1$, $q_n\to 1$ as $n\to\infty$. Then for each $f\in C_2^*[0,\infty)$ and all $\upsilon>0$, we can obtain $\lim_{n\to\infty}\sup_{x\in[0,\infty)}\frac{\left|R^*_{n,q_n}(f,x)-f(x)\right|}{\left(1+x^2\right)^{1+\upsilon}}=0$.

***Keywords***: $q$-calculus, $q$-Balazs-Szabados operators, weighted approximation.

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# On the vector-valued Banach algebra $l_p(X, A)$

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**Abstract**

Let $X$ be a non-empty set, $1 \leq p < \infty$, and $A$ be a commutative Banach algebra. Define $l_p(X, A)$ to be the set of all functions, $f: X \to A$, such that $\sum_{x \in X} \|f(x)\| < \infty$. This set forms Banach algebra under the usual operations and pointwise product. In this presentation, we present some basic algebraic properties of $l_p(X, A)$ inherited from $A$.

***Keywords:*** Commutative Banach algebra, vector-valued function.

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# A numerical approach for Singularly Perturbed Volterra Integro-Differential Equations on Bakhvalov-Shishkin Mesh

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**Abstract**

We consider an initial value problem with a first-order linear singularly perturbed Volterra integro-differential equation (SPVIDE). We first establish the results demonstrating the asymptotic behavior of the problem. We then construct a finite difference scheme for the problem on Bakhvalov-Shishkin mesh. To discretize the problem, we use the integral identities and the interpolating quadrature rules with remaining terms. The stability bound and the error estimates of the approximate solution are established. Further, we demonstrate that the scheme on Bakhvalov-Shishkin mesh is $O(N\^\{-1\})$ uniformly convergent, where $N$ is the mesh parameter. We also apply the scheme on a couple of test problems and observe that the numerical results reconcile with the theoretical results.

***Keywords:*** Singularly perturbed; VIDE; Difference schemes; Uniform convergence; Error estimates.

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# NONEXISTENCE OF GLOBAL SOLUTIONS FOR A KIRCHHOFF-TYPE WAVE EQUATION WITH TIME DELAY

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**Abstract**

In this paper, we are concerned with a Kirchhoff-type wave equation with time delay. Under appropriate conditions, we prove the nonexistence of global solutions. Generally, time delay effects arise in many applications and practical problems such as physical, chemical, biological, thermal and economic phenomena. Also, delay effects can be a source of instability.

***Keywords:*** Kirchhoff-type wave equation, nonexistence of global solutions, time delay.

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# EXPONENTIAL GROWTH OF SOLUTIONS FOR A VISCOELASTIC PLATE EQUATION WITH DISTRIBUTED DELAY TERM

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**Abstract**

In this paper, we consider a viscoelastic plate equation with distributed delay term. Under suitable conditions, the exponential growth of solutions have been discussed. Generally, time delay effects arise in many applications and practical problems such as physical, chemical, biological, thermal and economic phenomena.

***Keywords:*** Distributed delay term, exponential growth, viscoelastic plate equation.

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# Highly efficient numerical tools for solving nonlinear transport problems: Application to the coupled viscous Burgers' equations

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**Abstract:** We present an efficient numerical method for the simulation of nonlinear transport problems. The field of applications includes transport and dispersion of contaminants on large water surfaces such as the Mediterranean Sea. The space and time scales in this applications are very larges with complex patterns occur in both water flows and dispersion plumes. This multi-scale and multi-physics are very difficult to resolve using the conventional explicit Eulerian solvers because the time steps required in their simulations are limited with the well-known CFL conditions. In the current contribution we consider a coupled semi-Lagrangian scheme and cell-centered finite volume method to overcome these drawbacks associated with the Eulerian methods. The idea consists on using an implicit time stepping to integrate the governing equations along the characteristics in a finite volume discretization. The proposed method is CFL-free allowing for large time steps to be taken in the simulation. In this study we also examine the performance of a class of interpolation procedures using control volumes allocated in the vicinity of the element where the departure points belong. A third order Runge-Kutta scheme is implemented for time integration. We present numerical results for several test examples for coupled viscous nonlinear Burgers' equations. Comparison to the conventional finite element method is also carried out in the present study. The proposed techniques are also applied for numerical simulation of transport and dispersion of pollutants in the Mediterranean Sea.

**Keywords:** *Modified method of characteristics, Finite volume method, Transport-dispersion problems, Numerical simulation, Coupled Burgers' equations.*

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# A seaırd pandemic prediction model with fractional differential operator with analysis and numerical scheme

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**Abstract**

Nowadays, a work on a mathematical modeling about pandemic models are very popular in applied sciences. Recently many mathematical models have been considered and new methods have been used for approaching of these models. In this work we are interesting in mathematical modeling of a pandemic prediction model which is named as SEAIRD pandemic model. In this search the model is considered with fractional order differential operator of Caputo derivative. Also the existence and uniqueness result is verified. Finally numerical scheme is given with Adams-Bashforth method.

***Keywords:*** Fractional order derivative, existence and uniqueness, Adams-Bashforth numerical scheme.

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# A Half-Inverse Problem for a Quadratic Pencil of Sturm-Liouville Operator with Eigenparameter-Dependent Boundary Condition

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**Abstract**

In this work, we study on the following boundary value problem

$$L = L(q_0, q_1): \begin{cases} -y^{\Delta\Delta}(t) + \left(q_0(t) + 2\rho q_1(t)\right) y(\sigma(t)) = \rho^2 y(\sigma(t)), t \in \Omega, \\ y^{\Delta}(0) = 0, \\ \left(a\rho + b\right) y(l) = \left(c\rho + d\right) y^{\Delta}(l) \end{cases}$$

where $y^{\Delta}(t)$ denotes delta-derivative of $y^{\Delta}(t)$ and $\sigma : T \to T$ denotes forward jump operator, $\Omega = [0, a_1] \bigcup [a_2, l]$, for $a_2 > a_1$ and $a_1 + a_2 = l$; $q_0(t)$ and $q_1(t)$ is a real-valued continuous function on $\Omega$ and $\rho$ is the spectral parameter. Additionally, we assume that $ad - bc > 0$, $c \neq 0$. Together with $L$, we consider another operator $\tilde{L} = L(\tilde{q}_0(t), \tilde{q}_1(t))$ of the same form with different coefficients $\tilde{q}_0(t)$ and $\tilde{q}_1(t)$. Let $\{\rho_n : n \geq 0\}$ be the spectrum of both of $L$ and $\tilde{L}$. In this case, we obtain a new result for the half-inverse problem and prove that if $\tilde{q}_0(t) = q_0(t)$ and $\tilde{q}_1(t) = q_1(t)$ on $[a_2, l]$, then $\tilde{q}_0(t) = q_0(t)$ and $\tilde{q}_1(t) = q_1(t)$ on whole of $\Omega$.

***Keywords:*** Half-inverse problem, Closed sets, Boundary condition

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# Integrating Age-based Preventive Maintenance and Buffer Stock under Lease Contract using a Mathematical Model ; A Game Theoritic Approach

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**Abstract**
In continuous production units, lost production costs increase due to break downs, so the economic efficiency of these industries depends on proper preventive maintenance and maintenance policies to increase reliability and reduce equipment operating costs. In these industries, all systems, from the simplest to the most complex, require Scheduled maintenance to reduce the risk of failure. On the other hand, a subject that has already been extensively researched for years is the combined decision of preventive maintenance and safety stock. Most of the models reported in the literature implicitly assume that the company owns the manufacturing facility and in-house maintenance work is done. In this article, however, the acquisition of machinery through leasing is considered. The lessor who operates the machinery is responsible for servicing support under this lease contract. This contributes to the lessor's dispute with the lessee. We suggest a new cooperative and non-cooperative nash game theory for a production unit under a lease to overcome this conflict. The lessor's goal is to find the optimal scheduling of preventive maintenance actions which need to conduct and lease contract costs, and the lessee's goal is to determine the optimum buffer stock size that requires to be produced and the number of lease contract periods to minimize the overall cost to both sides over the lease period. In this scenario, where the analysis is performed from the lessor's and lessee's viewpoint, a mathematical formulation of game theory is required to model the decision problems. Concidering the duration of activities and the cost of inaccessibility as a time-dependent variable are other contributions. As the solution method, the genetic meta-heuristic algorithm is implemented and finally a numerical example is solved to evaluate the algorithm's performance.

***Keywords*****:** Preventive Maintenance- Safety Stock- Mathematical Formulation -Lease Contract- Game Theory

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# Soliton Solutions to the Resonant Davey-Stewartson Equations via Bernoulli sub-equation method

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**Abstract**

In this study, we use the Bernoulli sub-equation to study the analytical traveling wave solution of the (2+1)-dimensional resonant Davey-Stewartson system. In the beginning, Based on the wave transform, we convert the imaginary (2+1)-dimensional resonant Davey-Stewatson equation into nonlinear differential systems. The homogeneous balance method between the highest power terms and the highest derivative of the ordinary differential equation is authorized on the resultant outcome equation, and finally, the ordinary differential equations are solved to construct some new exact solutions. Different cases of solutions are constructed. Different values of physical constants to investigate the optical soliton solutions of the resulting system are used. The outcomes results of this study are shown in three dimensions graphically via Wolfram Mathematica Package.

***Keywords:*** The Bernoulli sub-equation method, Resonant Davey-Stewatson equations, Soliton solutions.

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# Indoor and Outdoor Navigation Application Development with Augmented Reality Technology

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**Abstract**

Augmented Reality (AR) technology is a technology that eliminates computer interfaces, enabling the real and virtual to be displayed in the same environment and to increase interaction with the real world. With the developing technology in recent years, a great development has been made in AR technology and it has become easier to use and develop, and it has begun to appear frequently today. The use of AR technology, which is used in many fields such as education, medicine, game industry, industry, tourism and advertising, is becoming widespread in the field of transportation. Navigation is a way-finding guide that guides the user along the specified route and reaches the desired location, and is a method of determining matters such as speed, location and direction during the journey. Navigation systems are essential in every period of human history, especially in ancient times. In today's changing and developing technology, there are navigation systems where the error rate is close to zero. Widely used 2D maps and navigation systems have been used together with AR technology to increase the user experience and add a different dimension to navigation applications. Today, there are many indoor and outdoor navigation applications that have adopted this approach. However, due to its complex structure and applicability, there are very few applications that offer indoor and outdoor navigation services together. In this paper, the results obtained from the indoor and outdoor navigation application developed using AR technologies will be presented. In this context, the application architecture and algorithm flows are supported by diagrams, the programming process and the technical knowledge are discussed in detail. With the application results developed, criteria such as the performance and usability of the AR navigation system are discussed.

***Keywords:*** Augmented Reality, Indoor Navigation, Outdoor Navigation, Unity, ARCore

# $2^{\Delta^{\widetilde{\alpha}}}$- STATISTICAL CONVERGENCE OF ORDER $\beta$ AND STRONGLY $2^{\Delta_{\beta}^{\widetilde{\alpha}}(p)}$-SUMMABILITY

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**Abstract**

In this study, by using the generalized difference operator, we introduce and examine the concepts of statistical convergence of order $\beta$ with the fractional order ($2^{\Delta^{\widetilde{\alpha}}}$ – statistical convergence of order $\beta$ ) and strongly Cesàro summability of order $\beta$ with the fractional order (strongly $2^{\Delta_{\beta}^{\widetilde{\alpha}}(p)}$ – Cesàro summability, $0<p<\infty$ ) for double sequences Also, we establish some inclusion relations between $2^{\Delta^{\widetilde{\alpha}}}$ – statistical convergence of order $\beta$ and strongly $2^{\Delta_{\beta}^{\widetilde{\alpha}}(p)}$ –Cesàro summability ( $0<p<\infty$ ).

***Keywords:*** Statistical Convergence, Difference Operator, Double Sequence, Cesàro Summability.

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# Investigation of Solitary Wave Solutions of Nonlinear Partial Differential Equation by Modified Exponential Function Method

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**Abstract**

In this study, some travelling wave solutions of the medium equal width equation were obtained using the modified exponential function method. The solution functions get include hyperbolic and trigonometric functions. 2D and 3D graphics of the obtained solution functions are drawn by determining the appropriate parameters.

***Keywords:*** The nonlinear partial differential equation; the medium equal width equation equation; the modified exponential function method.

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# Analysis of the Solution of Shallow Water Model

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**Abstract**

In this article, the travelling wave solutions of the shallow water like equation are obtained by using modified exponential function method. The resulting solution functions include hyperbolic and trigonometric functions. Two and three dimensional graphs were plot by determining the appropriate parameters for the solution functions.

***Keywords:*** The nonlinear shallow water like equation; the modified exponential function method.; travelling wave solutions.

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# A NEW SUBSTITUTION METHOD FOR NON-LINEAR BLOCK ENCRYPTION

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**Abstract**

Information technologies have gained more importance, especially in the Covid 19 pandemic period. In this process, the rate of online shopping increases, and different models are preferred by providing education and business life remotely. There are major grievances due to cyber-attacks made due to the security weakness in the databases of websites and servers.

sometimes related to each other, sometimes they operate independently. Block cipher algorithms are generally preferred for transactions requiring integrity checking. Also, Block encryption algorithms proposed by Shannon are based on diffusion and confusion methods. Scrambling is used to hide the relationship between the scrambled message and the public message while aiming to keep the traces in the spreading open message unnoticed in the encrypted message. Propagation and mixing occur by s-boxes displacement and linear transformation, respectively. In block cipher algorithms, more than one encryption step can be performed on the cycle of a block. Usually, the key to encrypt the message is used in each different step. The length of the key used in block encryption algorithms should be chosen to be stronger against the attacks to be made. The long preferred key makes it difficult to decipher the encrypted message against brute-force attacks. The number of steps performed in block cipher algorithms should be chosen appropriately. Thus, with the replacement processes, the encryption algorithm becomes both faster and stronger. Besides, the fact that the characters used in the password are different from each other is a very important factor in increasing the complexity. In this way, the clear message can be better protected against attacks.

With a new substitution method for the non-linear block cipher developed, the message is made meaningless by replacing every information recorded in the database of the servers with the help of a key. Since the steps of the existing block cipher algorithms are more (DES 16, BLOWFISH 16), the processing time increases. In a new substitution method for nonlinear block cipher developed, the low number of steps shortens the encryption time. Security is provided by making the recorded information more complex. Even if the attacks on servers and databases are successful, real and meaningful information will not be accessible because the information is encrypted incomprehensibly.

***Keywords:*** Block encryption, Database, Encryption

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# AN INTRODUCTORY STUDY ON THE RELATIONSHIP BETWEEN CATEGORY THEORY (CT) AND OBJECT-ORIENTED PROGRAMMING (OOP) PARADIGM

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**Abstract**

*Category Theory* (CT) serves as a common field of study among mathematicians who are specialized on diverse fields of *Mathematics*. As a matter of fact, what has led to the development of CT was the necessity of the formal definitions of some notions such as functors and natural transformations, which enable mathematicians to solve a problem in one branch of *Mathematics* using the tools of another branch. A functor between two categories assigns an object (and a morphism) of target category to each object (and to each morphism) of source category. While functions are the case between sets, functors relate two categories. In programming, functions in *functional programming* (FP) and methods in *object-oriented programming* (OOP) are used to accomplish a specific task to map inputs to outputs with some operations. When dealing with declaring the functions in FP, the classes covering data and methods (attributes/members in a class such as types and functions) are implemented in OOP by facilitating its principles such as (i) encapsulation, (ii) inheritance and (iii) polymorphism with composition. Moreover, while a category must comply with (1) identity, (2) composition and (3) associativity rules; *inheritance* and *composition* as OOP principles are 'is a' and 'has a' associations among objects, respectively. In this research, we therefore study the relationship between the concepts of CT with those from the OOP paradigm. Since, there are many implementations for the *applied* CT with FP by using *Haskell* and with OOP by using *Java* and such programming languages; a mathematician, computer scientist or a programmer can work on a category whose objects are sets (class of objects) and whose morphisms (class of morphisms) are functions between these sets.

***Keywords*:** Category theory (CT), morphism, functor, object-oriented programming (OOP), polymorphism, and composition.

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# MATHEMATICAL ANALYSIS OF THE M1 MOMENT IN ODD-MASS DEFORMED NUCLEI

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**Abstract**

In this study, we have analyzed the ground-state magnetic dipole (M1) moments of the deformed odd-A nuclei by a microscopic method called as Quasiparticle-phonon nuclear model (QPNM). The model operates with the strength function method, calculating the fragmentation of single-quasiparticle, one-phonon states, and the quasiparticle-phonon states over a large number of nuclear levels. Therefore, the wave functions of odd-mass nuclei consist of the sum of one-quasiparticle and one-quasiparticle ⊗ phonon [1].

Firstly, the wave function and the Hamiltonian of the system formed by nucleons are determined. Since the M1 excitations occur as a result of spin-spin interactions between nucleons, the spin forces are used in the analysis of M1 moments of the odd-A nuclei. Then, the mean value of the Hamiltonian is found via the wave function and a secular equation for the energies of an odd-A nucleus is derived using the Lagrange multipliers method. Secondly, we also obtain the set of equations for the coefficients, including the normalizations of the wave functions. Then, the intrinsic magnetic moment is derived from the expectation value of the z component of the magnetic dipole operator for a K state of an odd-mass nucleus [1].

Finally, we have compared the numerical results and the experimental data for the magnetic moments of the few nuclei, as an application.

***Keywords.*** *Magnetic dipole moment, QPNM, Spin-spin interaction*

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# On f-Deferred Statistical Convergence of Sequences of Sets

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**Abstract** In this paper, using an unbounded moduluf f we introduce the concepts of Wijsman f-deferred statistical convergence and Wijsman strong f-deferred Cesàro summability for sequence of sets. Furthermore some inclusion relations regarding Wijsman f-deferred statistical convergence and Wijsman strong f-deferred Cesàro summability are given.

***Keywords:*** Deferred Density, Deferred Cesàro Mean, Statistical convergence, Wijsman convergence.

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# Development of an Advanced Corporate Performance Management Suite

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## Abstract

Corporate Performance Management (CPM) is a common terminology that describes the methodologies, metrics, processes and systems used to monitor and manage the business performance of an enterprise by using Balanced Scorecard Methodology. Applications that enable CPM translate strategically focused information to operational plans and send aggregated results. These applications are also integrated into many elements of the planning and control cycle, or they address Business Activity Monitoring (BAM) or customer relationship optimization needs. CPM must be supported by a suite of analytical applications that provide the functionality to support these processes, methodologies and metrics.

CPMPlus is a CPM application which is providing Key Performance Indicator (KPI) management module for enterprise companies and government entities, dashboards, reports and graphical interfaces for organizational breakdown. CPMPlus deploys a single CPM solution across the entire organization, uses a single CPM solution to manage all financial processes, from close to disclosure, connects all applications and systems that house performance data to a single data source and gives users the ability to identify underlying trends in data, and see the impacts of business decisions across the organization.

CPMPlus has been developed with several tools including SQL Server, IBM TM1 Database, .Net, MDX, JavaScript and IBM Analytics. CPMPlus is based on the principle of the single point of truth, whereby each piece of information occurs only once within the system. It automatically covers all the operational and financial information to a comprehensive performance system including budget, actual, forecast, achievement and health status through index aggregation.

***Keywords:*** Organizational Development, Corporate Performance, Productivity

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# Forecasting Wind Energy Using a New Machine Learning Approach

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**Abstract**

With the decrease in the cost of electricity generation from wind, the use of wind energy is increasing in the world. The number of wind farms installed onshore and offshore has increased 75 times in the last 20 years. According to the data of the International Renewable Energy Agency, the total installed power of these facilities reached 564 gigawatt (GW) in 2018 from 7.5 GW in 1997. Also, in 2016, 16% of the total electricity produced by renewable energy facilities is provided by wind farms. However, due to the unpredictable and unstable nature of wind, it is difficult to forecast how much electricity wind turbines will generate. In this paper, its aimed to forecast ultra short term and short term wind energy using a three-step machine learning approach. In the first stage, using XGBoost, an error correction algorithm was developed by using the data received from the weather forecast providers and the data collected from the SCADA devices of the powerplants. In the second stage, a time series model was developed using Long-Short Term Memory (LSTM) and Multilayer Perceptron (MLP), and short term energy generation forecasts were created with this model. Finally, a new machine learning model has been developed using Support Vector Regressor (SVR) for ultra short term energy generation forecast, together with the the output of the time series model, and instant power data that can be collected from the IoT devices in the powerplants. As a result, it has been shown that the developed models produce acceptable error rates and can be used safely for the energy generation forecast of powerplants.

***Keywords:*** Wind Energy Forecast, Error Correction, Machine Learning

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# MACHINE LEARNING BASED CUSTOMER SEGMENTATION AND CAMPAIGN MANAGEMENT SYSTEM

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## ABSTRACT

In the fields of marketing and sales, identifying and segmenting the right customers for successful campaigns and recommending suitable campaigns to customers plays a significant role. Achieving successful results requires deep sector knowledge and analysis competence. However, these solutions which are mostly based on expert initiative cannot be standardized. The aim of this study is not only to create campaign recommendation system for customers but also customer segmentation system for campaigns and products. K-Means and K-Modes algorithms have been used for customer segmentation. On the other hand, Collaborative Filtering, Singular Value Decomposition (SVD), SVD++, Autoencoder and Deep Learning Matrix Factorization have been utilized to build several campaign recommendation systems. The dataset includes customer, product, sale and transaction discount information data. Mean Absolute Percentage Error (*MAPE*) has been utilized as the main error metric for evaluating the performance of the recommendation systems. It has been observed that the MAPE's of recommendation systems change between 13.26 and 20.49. It can be concluded that the developed recommendation systems yield acceptable error rates and these models can be used for identifying right customers and recommending appropriate campaigns to them.

***Keywords:*** Customer Segmentation, Recommendation System, Deep Learning

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# DEVELOPMENT OF A PERSONALIZED SEARCH ENGINE FOR E-COMMERCE SECTOR

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**Abstract**

Finding the desired product in e-commerce sector in the fastest and easiest way plays a vital role in customer satisfaction and revenue growth. Some interactive search engines have already been proposed in the literature and allow for text or visual queries [1-2]. Nevertheless, these studies focus on finding items aesthetically similar to the query without considering the query personalization aspects. Query personalization allows that user preferences are provided as a user profile separately from the query and dynamically decide how this profile will affect the query results. In this study, a personalized search engine is proposed, which is fed with the product data of the e-commerce site trendyol.com operating in the fashion-oriented retailing sector. More specifically, a search engine has been developed to recognize and help online shoppers find what they are looking for and discover a broader and more relevant range of products in the trendyol.com catalog. The index, search, and data collection infrastructures and a brand-based user-segmented product listing algorithm have been designed and implemented to realize the search engine. As the outcome of the study, a fashion-oriented and personalized site search has been enabled that successfully reveals products that have never been thought of before by directly associating the products the customers want. The results show that personalizing the search queries increase the odds of success. With the development of the personalized search engine, it is expected that Trendyol's revenues will grow in a short time through users visiting the site.

***Keywords:*** Search Engine; Information Retrieval; Query Personalization; Prediction

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# Random Forest Regression Combined with Minimum Reduncancy Maximum Relevance for Point of Sale Customer Lifetime Prediction

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**Abstract**

Feature selection methods make a significant contribution to machine learning algorithms in order to improve the operation speed and produce more accurate solutions by reducing the dimension of model and removing redundant and irrelevant features from model. Accordingly, in this study, Random Forest Regression combined with minimum Redundancy Maximum Relevance (mRMR) feature selection algorithm has been implemented in order to select optimal features used for prediction of the lifetime of customers that use point of sale (POS) machines. The dataset used in this study includes thirteen fetaures, which are maturity type, maturity interval, POS supplier company, city that the customer belongs to, total revenue of the customer, total transaction count through the POS, sector type of the customer, business type of the cutomer, sales channel, installment option, usage of cross product, POS type and discount type. mRMR has been applied on the dataset to rank the features and several customer lifetime value prediction models have been developed using different train-test split ratios. Symmetric Mean Absolute Percentage Error (SMAPE) has been utilized to assess the performance of lifetime prediction models. The results show that the lowest SMAPE (36.02 days) has been obtained for the model that includes all the features. It can be concluded that Random Forest Regression is a viable method that can be used to predict the lifetime of customers that use point of sale (POS) machines within acceptable error rates.

***Keywords:*** Random Forest Regression, Feature Selection, Customer Lifetime

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# Developing Turkish Sentiment Analysis Models Using Machine Learning and E-commerce Data

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**Abstract**

With the increment of the Internet usage, there has been big increase in the access and interaction of users in social media, blogs, forums and criticism sites recently. With social media, access to large amount of data on various products, services, social and political events is provided. Important feedback about products and services can be obtained as a result of analyzing such data. In this paper, it is aimed to determine the sentiments expressed via texts on social media using machine learning methods. As a result of initial research, it is determined that the best case in which texts and emotions match was the product reviews and ratings used on e-commerce web sites. Reviews on different products along with review-scores from an e-commerce web site have been converted into a table to be used in the machine learning based sentiment analysis models. Reviews have been classified into three groups as positive, negative and neutral using the review-scores. Considering this claim, Turkish sentiment analysis models were developed using different machine learning methods including support vector machine (SVM), decision tree and logistic regression. Cross-validation results on an independent test data taken from the same e-commerce web site show that SVM based sentiment analysis model outperforms the other models. The overall accuracy of the SVM based sentiment analysis model is found to be 88%. Also, it is seen that the prediction accuracy of positively labeled comments, the negatively labeled comments and neutrally labeled comments have been 93%, 86% and 72.6%, respectively.

***Keywords:*** Sentiment Analysis, Natural Language Processing, Machine Learning

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# A New Numerical Approach for Generalized Fisher's Equation

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**Abstract**

This study presents a numerical approach for the generalized Fisher's equation as a result of the application of the splitting techniques using the finite element collocation method combined with the B- spline. For this purpose, after the proposed problem is divided into two sub-equations containing the time-dependent derivative, these sub-equations are converted to the ordinary differential equations system (ODEs) with the help of the cubic B-spline finite element collocation method. After these operations, the ODEs obtained by sub-equations are solved numerically with the splitting techniques via the fourth order Runge-Kutta (RK-4) algorithm in terms of more accurate, easier computer applicability. To indicate the effectiveness of the presented method, the error norms $L_2$ and $L_\infty$ are computed and their results are exhibited in graphics as well as tables. These new numerical results show that the method we suggest is sufficiently suitable and consistent for various types of nonlinear partial differential equations.

***Keywords:*** Generalized Fisher's equation, B-spline collocation method, Runge–Kutta method, splitting techniques.

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# CONFORMAL BI-SLANT $\xi^{\perp}$-SUBMERSIONS FROM SASAKIAN MANIFOLDS

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**Abstract**

In this paper, we investigate conformal bi-slant $\xi^{\perp}$-submersions from Sasakian manifolds onto Riemannian manifolds as a generalized bi-slant $\xi^{\perp}$-Riemannian submersions. We investigate the integrability of distributions and obtain necessary and sufficient conditions for the maps to have totally geodesic fibers.

***Keywords:*** Sasakian manifold, conformal submersion, conformal bi-slant $\xi^{\perp}$-submersion.

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# M- STURM-LIOUVILLE PROBLEM WITH MODIFIED COULOMB POTENTIAL

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**Abstract**

In this study, we consider M-derivative Sturm-Liouville problem having modified Coloumb potential as follows:

$$-D_M^{\alpha,\gamma} D_M^{\alpha,\gamma} y(x) + \alpha^2 \left( \frac{A}{x^\alpha} + q(x) \right) y(x) = \alpha^2 \lambda y(x) \tag{1}$$

$$y(a) \cos \alpha + D_M^{\alpha,\gamma} y(a) \sin \alpha = 0 \tag{2}$$

$$y(b) \cos \beta + D_M^{\alpha,\gamma} y(b) \sin \beta = 0 \tag{3}$$

where $\frac{y(x)}{x^\alpha} \in C^{2\alpha}[0, \pi]$.

Using the Laplace transform, we obtain the representation of solution of problem (1)-(3).

***Keywords:*** M-derivative, Sturm-Liouville problem, Laplace transform, Coloumb Potantial.

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# Numerical Solution of Volterra Integral Equations with Hosoya Polynomials

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**Abstract:** In this study, Volterra integral equation is solved by Hosoya Polynomials. The solutions obtained with these methods were compared on the figure and table. And error analysis was done. Matlab programming language has been used to obtain conclusitions, tables and error analysis within a certain algorithm.

**Keywords:** Integral Equations , Volterra Integral Equations, Hosoya Polynomial, Path Graphs.

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# WAVE SIMULATIONS IN LIQUIDS CONTAINING GAS BUBBLES

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**Abstract**

In this paper, we apply an efficient method which is the sine-Gordon expansion method to present new wave simulations of a governing model. We consider the conformable (3+1)-dimensional Kudryashov-Sinelshchikov equation describing wave propagation in bubble gas liquid. We have various type solutions such as dark, bright and periodic wave. Finally, some figures are showed physical behaviours of obtained solutions with by 3D,2D and contour surfaces.

***Keywords:*** The sine-Gordon expansion method; the conformable (3+1)-dimensional Kudryashov-Sinelshchikov Equation; travelling wave solutions.

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# A two-groups Covid 19 epidemiological model: Stability analysis and numerical simulations using neural networks

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**Abstract**

In this paper, we investigate the asymptotic behavior of a two groups Covid 19 epidemiological model. The basic reproductive number and the equilibrium points are determined. Conditions of stability are established. The model illustrates the influence of mobility on the qualitative behavior of the disease. The paper proposes the use of recent approaches of neuro ordinary differential equations in order to solve some models of ordinary differential equations in epidemiology, and to illustrate different scenarios for various parameters values. Finally, we carry out numerical simulations and we present some promising results.

***Keywords:*** Epidemiological model - Stability analysis – Neuro ordinary differential equations.

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# ON EXACT SOLUTIONS OF WICK-TYPE STOCHASTIC EXTENDED KDV EQUATION

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**Abstract**

In recent years, a lot of attention has been given to the study of SPDEs with Lévy noise, without Gaussian component. Biorthogonal decomposition plays the role of the main tool of white noise theory which is an expansion of the Winner-Ito chaos. The white noise analysis approach to quantum physics was originally applied to a wide range of other applications.

In this paper, the method of F-expansion with the aid of Hermit transformation are used to find a series of exact elliptic function solutions to the nonlinear KdV extended Wick-type stochastic equation with a random term of Levy white noise type. By means of these methods and with the help of Maple we get solitary wave solutions and determined for KdV extended Wick-type stochastic equation. 2D, 3D, and contour graphs have been drawn by giving special values to the constants in the solutions via computer software. Moreover, by considering different random values to the noise, the effect of the noise on the wave-forms has been exhibited. The obtained results have been discussed in detail

***Keywords:*** Stochastic Extended KdV-equation, Wick-type stochastic, white noise, Brownian motion, Exact solution, F-expansion method , Hermite transformation.

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# LUCAS DIFFERENCE SEQUENCE SPACES OF FRACTIONAL ORDER

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**Abstract**

The main goal of this study is to define and investigate the structure of fractional order Banach set with the help of Lucas matrix whose terms are the well-known Lucas numbers. To do this, we first compund the concepts of Lucas matrix and fractional order difference operator. After then, we examine topological and geometrical structure of the defined space.

***Keywords:*** Lucas numbers, sequence space, modulus of convexity, fractional order

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# CHIRPED W-SHAPED OPTICAL SOLITONS OF THE RANGWALA-RAO EQUATION

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**Abstract**

In this study, the exact nonlinearly chirped soliton solutions are derived from a special nonlinear Schrödinger equation known as Rangwala-Rao equation which are of interest in describes plasma physics, wave propagation in nonlinear optical fibres, Ginzburg-Landau theory of superconductivity. Propagating chirped soliton solutions for Rangwala-Rao equation investigated by application of the ansatz method.
***Keywords:*** Ansatz method, Chirped soliton, Exact solution

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# Regarding new wave distributions of the nonlinear integro-partial ITO differential and fifth-order integrable equations

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**Abstract**

This paper applies a powerful scheme, namely, Bernoulli sub-equation function method to the some nonlinear partial differential equations. We obtain many new travelling wave solutions such as mixed dark-bright soliton, exponential and complex domain. Under the suitable choosing of the values of parameters, we simulate the wave behaviours of the results obtained in the paper in terms of 2D, 3D and contour surfaces.

***Keywords***: Integro-partial differential equation, Fifth-Order integrable model, analytical method, Rational function solution, Complex Solution, Contour surface, Travelling wave solutions.

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# STABILITY ANALYSIS FOR THE VOLTERRA DELAY INTEGRO-DIFFERENTIAL EQUATION

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**Abstract**

In this paper stability inequalities for the linear non homogeneous Volterra delay integro-differential equation (VDIDE) is being established. Test example is encountered to show the applicability of the method and to confirm the predicted theoretical analysis.

***Keywords:*** Integro-differential equation; Delay differential equation; Stability inequality.

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# BIDIRECTIONAL ASSOCIATIVE MEMORY TYPE FRACTIONAL ORDER IN ANTI-PERIODIC SOLUTIONS

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## Abstract

This article discusses state prediction for a class of bidirectional associative memory (BAM) neural networks. Using the inequality theory technique and the Lyapunov functional method, some sufficient conditions were provided to ensure the existence of anti-periodic solutions and their exponential stability globally for discrete-time bidirectional neural networks. An example is provided to demonstrate the accuracy of the analysis.

***Keywords***: bidirectional associative memory, inequality technique, anti-periodic, Lyapunov functionality, fractional differential equation, delays

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# Comparison of Modal Assurance Criteria with Fuzzy Similarity to Detect Damage to Structures

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**Abstract:**

Up today, many methods have been tried to detect and locate damage in structures. Most of these approaches focus on statistical analysis for damage identification considering only random uncertainties and without parsing the data at all, so the results are not too careful. In this work, we will compare the modal assurance criterion from the old static method that allows accommodating other types of uncertainties due to ambiguity, vagueness, and fuzziness which are statistically non-describable and fuzzy similarity.

**Keywords:** Modal Assurance Criterion, Fuzzy Similarity, Damage Detection, Structural Damage Pattern

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# OPTICAL SOLUTIONS OF CONFORMABLE EXTENDED CALOGERO-BOGOYAVLENSKII-SCHIFF EQUATION

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**Abstract**

We implement an analytical approach which is the sine-Gordon expansion method to the extended Calogero–Bogoyavlenskii–Schiff equation in conformable sense. By this way, we find out some optical soliton solutions which contain hyperbolic, trigonometric, periodic functions. We give 3D,2D and contour plots of waves for better understand physical behaviors in engineering and physics.

***Keywords:*** The sine-Gordon expansion method; the conformable extended Calogero–Bogoyavlenskii–Schiff equation; optical soliton solutions.

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# ON NUMERICAL SOLUTION OF A NONLINEAR SCHRODINGER EQUATION INCLUDING MOMEMTUM OPERATOR

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**Abstract**

In this paper, we study the numerical solution of a nonlinear Schrödinger equation. Schrödinger equation is previously analyzed by various methods such as Adomian decomposition method, Homotopy Perturbation Method, Homotopy Analysis Method, Variational Method, Galerkin's Method, Finite Difference Method [1-5]. In the paper, we apply the finite difference method to a nonlinear Schrödinger equation which contains a momentum operatör. We obtain an estimate for the solution of difference scheme. Then, using the estimate, we prove that the difference scheme is unconditionally stable. Finally, the convergence of the scheme is shown.

***Keywords:*** Schrödinger equation, Finite difference method, Stability, Convergence.

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# Numerical Solution of the coupled Burgers' equation by Trigonometric B-spline Collocation Method

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**Abstract**

In the present study, the coupled Burgers' equation is going to be solved numerically by presenting a new linearization technique based on collocation finite element method in which trigonometric cubic and quintic B-splines are used as approximate functions. In order to support the present study, three test problems given with appropriate initial and boundary conditions are studied. The newly obtained results are compared with some of the other published numerical solutions available in the literature. The accuracy of the proposed method is discussed by computing the error norms $L_2$ and $L_\infty$. A linear stability analysis of the approximation obtained by the scheme has been investigated.

***Keywords:*** Finite Element Method, Collocation Method, Coupled Burgers' Equation, Trigonometric B-splines

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# Coefficient Inequalities for A Subclass of Bi-Univalent Functions Defined by Pell-Lucas Polynomials

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**Abstract:** In this study, we investigate a certain subclass of bi-univalent functions defined by Pell-Lucas polynomials. For functions belonging to the defined subclass, we then derive coefficient inequalities and Fekete-Szegö inequalities.

**Keywords:** Analytic functions, Bi-univalent functions, Coefficient inequalities, Fekete-Szegö inequality, Pell-Lucas polynomials.

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# Design of neuro swarming heuristic for a class of singular singularly perturbed two-point boundary value problems

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**Abstract:** In the present study, a neuro-evolutionary scheme is presented for solving a class of singular singularly perturbed boundary value problems (SSP-BVPs) by manipulating the strength of feed-forward artificial neural networks, particle swarm optimization (PSO) and interior-point algorithm (IPA). Using the approximation capability of universal function of ANNs, the mathematical modeling of SSP-BVPs is made in mean square sense. Initially, the global search technique PSO is used to approve the design parameters and then local search technique IPA is applied for local refinements. Two variants of the SSP-BVPs along with four cases are tested to authenticate the performance of the designed scheme. Moreover, comprehensive simulations are presented to authenticate the efficiency of the designed approach in terms of robustness, convergence and accuracy.

**Keywords:** Singular singularly perturbed; Particle Swarm Optimization; Artificial Neural Network; Statistical Analysis; Interior-Point

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# On a Sequence of Linear Positive Operators Associated with *k*-Beta Function

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**Abstract**

Approximation theory is an exciting field that deals with functions which can be approximated as efficiently as possible from simpler functions. One of the most important aspects of this theory is linear positive operators. With the remarkable theorem due to Weierstrass [3] and the fundamental convergence theorem given by Korovkin [2], several new operators have been constructed and their various convergence properties have been investigated in different function spaces by many researchers from past to present.

Our present investigation is motivated essentially by the recent publication given by Bhatt et al. [1]. In this talk, we present a new modification of linear positive operators constructed by using *k*-Beta function. We discuss the rate of convergence of the operators by means of the modulus of continuity and also we investigate the approximation theorem in Lipshitz class. Moreover, we prove the Voronovskaja type theorem which is essential to inquire about the pointwise convergence and finally we present some illustrative graphics to compare the approximation with the well-known linear positive operators.

***Keywords:*** Korovkin Theorem, *k*-Beta function, modulus of continuity.

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# A NUMERICAL TEST FOR A NEAR WALL MODEL

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**Abstract**

It is difficult to find appropriate boundary conditions for the flow averages which depend on the behavior of the unknown flow near the wall. Large eddy simulation (LES) is one of the promising approaches. LES seeks to predict local spatial averages $\bar{u}$ of the velocity u of the fluid. The LES equations are solved over moderate time intervals in some applications. But the main problem is associated with modeling near wall laminar in complex geometries. Therefore, an important problem in LES is to find appropriate boundary conditions for the flow averages which depend on the behavior of the unknown flow near the wall. Inspired by the works of Navier and Maxwell [1,2], the boundary conditions are imposed on the wall. In this study, the appropriate friction coefficient for 2-D laminar flows is computed and existing boundary layer theories are used to improve numerical boundary conditions for flow averages. The slip with friction and penetration with resistance boundary conditions are considered. Numerical tests on two dimensional channel flows across a step using this boundary condition on the top and bottom wall as well as on the step are performed.

***Keywords:*** Boundary Layers, Laminar, Near Wall Models (NWM), Box Filter, Uniform Suction.

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**The Solution of The Linear Delay Differential Equations with Aboodh Transform**

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**Abstract:**

In this paper, we apply Aboodh transform to solve linear delay differential equations. Firstly, The basic properties of the Aboodh Transform are given. Secondly, Existence of the Aboodh transform proved. Then, the two linear delay differential equations are solved by Aboodh transform. This means that Aboodh transform is a powerful tool for solving linear delay differential equations.

**Keywords:** Linear Delay Differential Equations, Aboodh Transform, Existence of the Aboodh Transform

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# Investigation of the Wave Solutions of the Double-Chain DNA Model by the Modified Exponential Function Method

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**Abstract**

In this study, the wave solutions of the (2 + 1)-dimensional equation of the double-chain DNA model which is formulated mathematically as a nonlinear partial differential equation is found by using the modified exponential function method. The solution functions, obtained as a result of the all calculations, are revealed to be hyperbolic, trigonometric and rational functions. By determining the appropriate parameters for the obtained solution functions, two and three-dimensional graphs, contour graphs and density graphs are shown and these graphs represent the characteristic behavior of the solution functions.

***Keywords:*** the (2 + 1)-dimensional equation of the double-chain DNA model; the wave solutions; the modified exponential function method.

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# STABILITY AND BIFURCATION ANALYSIS IN A DISCRETE-TIME PREY-PREDATOR MODEL, AND CHAOS CONTROL

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**Abstract**

Lotka - Volterra systems are considered as a building block in the development of more realistic mathematical models. In this study, we investigate the qualified behavior of the discrete-time system obtained with the Euler scheme. The conditions of existence for bifurcations are discussed by using bifurcation theory and center manifold theorem. Chaos control of the discrete-time system is also performed through chaos control strategy.

***Keywords:*** bifurcation, chaos control, stability, prey-predator system

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# INVERSE PROBLEM FOR THE GRAPH INVARIANTS

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**Abstract**

Graph invariant, namely topological index, based on degrees or distances in graphs is the number related to graph. Inverse problem is about for which positive integers there is a graph with these graph invariants of that values. In this work, we consider inverse problem for co-PI index and give a note for Sombor index.

***Keywords:*** *co-PI index, Graph Theory, Degree, Distance, Inverse problem, Sombor indices.*

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# A mathematical model of Covid-19 spread with piecewise modeling

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**Abstract**

In this study, we consider a mathematical model of Covid-19 outbreak with piecewise differential and integral operators suggested by Atangana and Seda. They showed that these operators are useful to model real world problems having cross-over behaviors. Indeed, considering that the spread of the coronavirus exhibits crossover behavior in some countries, it becomes clear how important the use of these operators is. Based on these reasons, we consider the model with these operators in this study. In addition to detailed analysis for this model, diagrams, charts and predictions for the number of cases in some countries will be presented. Finally, numerical simulations are depicted for numerica solution of the considered model.

***Keywords:*** Covid-19 model, piecewise differential and integral operators, numerical scheme.

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# A New Treatment of the Finite Difference Method for 2-interval Sturm-Liouville Problems

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## Abstract

Finite Difference Method (FDM) and its various genelalizaton are intended for 1-interval initial and/or boundary value problems for ordinary and partial differential equations. In this study, we developed a new modification of FDM to solve not only 1-interval problems but also 2-interval differential equations with additional transmission conditions at one internal point of transition. Namely, we investigate the following 2-interval boundary value problem, consisting of the second order differential equation

$$xy^{"} - y^{'} + 4x^3y = 0,$$

on two disjoint intervals $[-1, 1)$ and $(1, 3]$, separated boundary conditious at the end points $x = -1$ and $x = 3$, given by

$$y(-1) = 0, \quad y(3) = 2,$$

and transmission conditions at an internal transition point $x = 1$, given by

$$y(1-0) = 2y(1+0), \quad y'(1-0) = 3y'(1+0).$$

In fact, we have two differential equations for left and right solutions. Additional transmission conditions are given in terms of limit values of the left and right solutions at the transition point.The calculated numerical solutions are compared with the exact solutions. Graphical illustrations of the obtained numerical and exact solutions are also presented.

***Keywords:*** Finite difference method, transmission conditions.

# TRAVELING WAVE SOLUTIONS OF NONLINEAR PARTIAL DIFFERENTIAL EQUATION

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**Abstract**

In this study, authors employed the an expansion method to obtain the new traveling wave solutions of nonlinear partial differential equation. With this expansion method, traveling wave solutions have been obtained. Many different processes were encountered during the acquisition of these solutions. These difficult stages can be easily reached with the help of computer package program. By giving arbitrary values to the constants in the solutions obtained, the state of the wave at any time of is presented with 3D, contour and 2D graphs. The results obtained reveal that the applied method is a good alternative for accurate, effective, easy to apply and nonlinear partial differential equations.

**Keywords:** An expansion method, Nonlinear partial differential equation, traveling wave solution.

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# A NEW TREATMENT OF THE FINITE DIFFERENCE METHOD FOR 2-INTERVAL STURM-LIOUVILLE PROBLEMS

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**bstract:**

Finite Difference Method (FDM) and its various genelalizaton are intended r 1-interval initial and/or boundary value problems for ordinary and partial fferential equations. In this study, we developed a new modification of FDM to lve not only 1-interval problems but also 2-interval differential equations with lditional transmission conditions at one internal point of transition. Namely, e investigate the following 2-interval boundary value problem, consisting of the cond order differential equation

$$xy'' - y' + 4x^3 y = 0,$$

n two disjoint intervals $[-1, 1)$ and $(1, 3]$, separated boundary conditious at the nd points $x = -1$ and $x = 3$, given by

$$y(-1) = 0, \quad y(3) = 2,$$

nd transmission conditions at an internal transition point $x = 1$, given by

$$y(1-0) = 2y(1+0), \quad y'(1-0) = 3y'(1+0).$$

n fact, we have two differential equations for left and right solutions. Additional ansmission conditions are given in terms of limit values of the left and right lutions at the transition point.The calculated numerical solutions are compared ith the exact solutions. Graphical illustrations of the obtained numerical and xact solutions are also presented.

***Keywords:*** Finite difference method, transmission conditions.

# Blockchain for Foreign Trade

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**Abstract**

There is a long history of studies conducted for correspondence in import and export transactions in accordance with certain standards. For example, countries in the EU economic group, including Turkey, prepare their customs declaration in accordance with EDIFACT which enabled documents to be shared between relevant organizations and directly transferred to information systems.

Blockchain offers new possibilities for import and export transactions. The blockchain will provide document sharing among partners and provide significant credibility by keeping a record of the shared documents. As the leading customs consultancy firm (UGM) in Turkey, we have started to work to carry out customs clearance transactions on the block chain. In order to establish the block chain, the necessary technical infrastructure must be established, as well as the partners to participate in the chain. For this purpose, a bank, a logistics firm, an insurance company and an IT firm were brought together by UGM. The team created works on the determination of documents as customs clearance, work flow, processes, standards, information security, time stamp, electronic signature and infrastructure. The results of this collaboration will be presented in this paper.

Keywords: Blockchain, Foreign Trade, Information Security, Phrase, Time Stamp

# ITÔ STOCHASTIC DİFFERENTIAL EQUATION MODELING FOR BITCOIN DATA

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**Abstract**

In this study, Bitcoin data is examined by Stochastic Differential Equation Modeling (SDEM). At first, the parameters of SDE established for the given Bitcoin data are estimated by using maximum likelihood estimation method. Then, we have obtained reasonable Stochastic Differential Equation (SDE) based on the Bitcoin data. Finally, by applying Euler-Maruyama Approximation Method trajectories of SDE according to the fixed time are achieved. The performances of trajectories are established by Chi-Square criteria. The results are acquired by using statistical software R-Studio.

***Keywords:*** Itô stochastic differential equation, Euler-Maruyama approximation method, Maximum likelihood estimation method, Bitcoin data

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# THE EFFECT OF DIFFERENT SEARING DEGREES ON THE COLOR PROPERTIES OF COOKED MEAT

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## Abstract

The color and appearance of meat and meat products are significant parameters affecting consumer preference and purchase decisions (Suman and Joseph, 2013). Therefore, color evaluation is an essential part of meat research, product development, and troubleshooting (Hunt and King, 2012). A typical instrumental technique used for evaluating the color features of meat is colorimeter that defines the meat color in terms of the values $L^*$ (lightness), $a^*$ (redness), and $b^*$ (yellowness), commonly referred to as the Hunter values (King and Whyte, 2006). The color of meat revolves around myoglobin, the primary red pigment in meat, and the cooking process causes denaturation of myoglobin. Heat-induced myoglobin denaturation is responsible for the dull-brown color of cooked meats (Hunt and King, 2012). Searing is a technique of cooking food surfaces at high temperatures until a brown crust is formed. It has been reported that crust that forms on the surface of the meat as a result of searing is beneficial to the appearance of meat (Barber and Broz, 2011). In the present study, the effect of different searing degrees on the color properties of cooked meat was investigated. The meat slices (5 cm) were seared until their internal temperature reached 20 °C, 30 °C, and 40 °C. Then samples were cooked at 180 °C in the oven until their internal temperature reached 71 °C. Control group samples were cooked in the oven without searing. The different searing degrees had a very significant ($P<0.01$) effect on the $L^*$ and $a^*$ values of cooked meat, while searing had no significant effect ($P>0.05$) on the $b^*$ value. The $L^*$ value of the samples varied between 34.62 and 48.90. The highest and lowest $L^*$ values were determined in the control group samples and samples seared up to 40 °C internal temperature, respectively. $a^*$ value varied between 7.48 and 12.88. While the highest $a^*$ value was determined in the control group samples, no statistical difference was observed between the samples seared at different degrees.

***Keywords:*** *Meat, searing, cooking, color*

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# NEW SOLUTİONS OF (3+1)-DİMENSİONAL BREAKİNG SOLİTON EQUATİON AND (3+1)-DİMENSİONAL SHALLOW WATER-LİKE EQUATİON

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**Abstract**

In this article, _rst, we expressed the (3+1)-D breaking soliton equation and (3+1)-D shallow water-like equation and then explained the $\exp(-\gamma(\zeta))$-Expansion method we will apply to these equations. In addition, by using wave transformation, partial di_erential equations were transformed into ordinary di_erential equations and solutions were found. New solutions are attained by employing the $\exp(-\gamma(\zeta))$-Expansion Method for the (3+1)-D breaking soliton equation and (3+1)-D shallow water-like equation. Equations include exponential, hyperbolic, and rational solutions. Also the solutions are singular. All solutions obtained for arbitrary values of the parameters included in the solution were drawn in plot3D, Contourplot, and Discreteplot from figure 1 to figure 10. In the hydraulic engineering of the equations, the shapes we will draw will help us to calculate the water on the shore, the depth, the state of the wave in the water and to see the state of the wave. New solutions have been given to the literature for the (3+1)-D breaking soliton equation and (3+1)-D shallow water-like equations. These results show that water ooding from coastal waves is an important point for use in hydraulic engineering.

***Keywords:*** $\exp-\gamma(\zeta))$- Expansion Method, Travelling Wave Solutions, (3+1)-dimensional Breaking Soliton equation, (3+1)-dimensional Shallow Water-like equation.

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# DISCRETE GRADIENT FIELD ON POINT SET SURFACE VIA TIME SCALE CALCULUS

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**Abstract**

The theory of time scales calculus have long been a subject to many researchers from different disciplines. Beside the unification and the extension aspects of the theory, it emerge as a powerful tool for mimetic discretization process. In this study, we present a framework to find gradient vector fields of discrete point sets in $\mathbb{R}^3$ by using symmetric differential on time scales. A surface parameterized by the tensor product of two time scales can be analogously expressed as the vertex set of non-regular rectangular grids. If the time scales are dense, then the discrete grid structure vanishes. If the time scales are isolated, then the further geometric analysis can be executed by using symmetric dynamic differential. Our results indicate that the method we present has good approximation to gradient vector fields of parameterized surfaces.

***Keywords:*** Symmetric derivative, Discrete differential geometry, Discrete gradient field

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# ON SOME INTEGRAL INEQUALITIES VIA GENERALIZED PROPORTIONAL FRACTIONAL INTEGRAL OPERATORS

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**Abstract**

In this paper, we have provided some new integral inequalities for integrable and positive valued functions by using the generalized proportional fractional integral operators. To prove the main findings, we have consider some classical inequalities and basic definitions.

***Keywords:*** Generalized proportional fractional integrals, Young inequality, integral inequalities.

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# An application of absolute matrix summability

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**Abstract**

In this paper, we extended a theorem proved by (W.T. Sulaiman, Extension on absolute summability factors of infinite series, Journal of Mathematical Analysis and Applications, 322, 1224-1230, 2006) dealing with absolute weighted arithmetic mean summability factors of infinite series to $\varphi-\left|A, p_n\right|_k$ summability method by using a quasi f-power increasing sequence.

***Keywords:*** Summability; infinite series; absolute matrix summability.

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# Osculating direction curves of non-lightlike curves in $E_1^3$

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**Abstract:** In this study, it's introduced osculating direction curves and osculating donor curves of timelike and spacelike curves in $E_1^3$ Minkowski Space and obtained characterizations and some results for these curves.

**Keywords:** Associated curve, osculating direction curve, osculating donor curve

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# KINEMATIC EQUATIONS OF A MECHANISM IN LORENTZ 3-SPACE

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**Abstract**

We design a mechanism in Lorentz 3-space. The Denavit-Hartenberg representation and the homogeneous coordinates are used for construsting the mechanism. Forward kinematic equations are written for 5R mechanism.

***Keywords:*** Denavit-Hartenberg parameters, Kinematics, Lorentzian 3-space.

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# Investigation of Exact Wave Solutions of the Exponential Nonlinear Klein-Gordon Equation

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**Abstract:** In this study, exact traveling wave solutions of Klein-Gordon Equation with an exponential nonlinearity using the new function method. Two-three dimensional, density, and contour graphs of the obtained solution function were drawn by selecting the appropriate parameters. The solutions of the obtained graphs and equations were found with the help of the Mathematica package program.

**Keywords:** New function method; Klein-Gordon Equation with an exponential nonlinearity.

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# Analysis of Wave Solutions of The Nonlinear Model Equation with New Function Method

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**Abstract**

In this study, the Sinh-Poisson equation's wave solution has been obtained by using the new function method. When the solution function of this nonlinear model equation is analyzed, it is seen that it is the function that includes the periodic behavior model. Two and three-dimensional density and contour graphs were obtained by determining the parameters suitable for the solution function visually of this periodic behavior model.

***Keywords:*** The Sinh-Poisson equation; the new function method.

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# PROVIDING AN ALTERNATIVE METHOD FOR COMBINING ODDS RATES

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"Meta Analysis" was first used by Glass in 1976 to combine research in the field of education, and then it started to be applied in many areas. Meta analysis is a method of combining the results of multiple studies that are independent from each other and performing the statistical analysis of the research results obtained. Meta-analysis can be applied to quantitative studies. For example; controlled clinical trials, observational studies, etc.

In our study, 2x2 type data were produced by using the "R" program by taking clinical research as an example. A data group was formed by combining the data produced in this way, and the odds ratios calculated for the 2x2 data sets in this data group were combined with the general variance-based method and Mantel Haenzel method. Afterwards, the measurement of the total effect of the distribution of the case group on the control group was combined using the $KL$ ($V$, $K$) statistic and it was concluded that this statistic could be used as an alternative to the odds ratio.

**Key Words:** Meta analysis, Mantel Haenszel Method, Odds ratio, Kullback Leibler Divergence.

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# On Survey of the Some Wave Solutions of the Nonlinear Schrödinger Equation in Infinite Water Depth

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**Abstract:** In this research study, we use two different analytical scheme which are the sine-Gordon expansion method and the modified $\exp(-\Omega(\zeta))$-expansion function method to construct novel exact solutions of the nonlinear Schrödinger equation, describing gravity waves in infinite deep water, in the sense of conformable derivative. After getting various travelling wave solutions, we plot 3D, 2D and contour surfaces to present behaviours obtained exact solutions.

**Keywords:** The Sine-Gordon Expansion Method, The modified $\exp(-\Omega(\zeta))$-expansion function method, Conformable Derivative, Nonlinear Schrödinger Equation, Traveling Wave Solutions.

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# On Texture Graph

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**Abstract**

It is known that topological graphs by combining graphs to topologies, and bitopological graphs are obtained by combining graphs to bitopologies. In this paper, graphs which we called Ditopological Texture Graph, have been obtained by matching the diagrams with the ditopologies. Also, some properties of ditopological texture digraphs are investigated.

***Keywords:*** Texture space, ditopology, graph

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# NEW TRENDS IN BLOCKCHAIN TECNOLOGY AND A GLANCE AT CRYPTOCURRENCIES

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**Abstract**

Because of the Covid 19 pandemic all over the world, people are more involved in digital life style in various aspects such as education, communication, work-related tasks and investments. Blockchain technology, which is essentially a decentralized and distributed database is used as a ledger to form a peer-to-peer network of continous flow of transactions, provides a foundation for an increasing number of digital assets where public key cryptography and hash functions play a major role. In this paper, we study applications of blockchain in various fields and cryptocurrencies.

***Keywords:*** Blockchain, cryptocurrencies, hash function, public key cryptography.

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# Construction of Soliton Solutions for Chaffe–Infante Equation

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**Abstract**

In this article, we investigated the soliton solutions of the Chafee–Infante equation. In accordance with this purpose, we used sine-Gordon expansion method, which is one of the solution methods of nonlinear partial differential equations. Graphical representation of the obtained results of the specified equation is made using Wolfram Mathematica 12 for certain values and thus the conformity of the founded results has been demonstrated.

***Keywords:*** Chafee–Infante equation, sine-Gordon expansion method, soliton solutions.

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# Novel Solutions of Perturbed Boussinesq Equation

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**Abstract**

In this article, we have worked the perturbed Boussinesq equation. We have applied the GKM, which has been used by many researchers recently, to the perturbed Boussinesq equation. Thus, we have acquired some new soliton solutions of the perturbed Boussinesq equation. Also, we have drawn some 2D and 3D surfaces of these obtained results by using Wolfram Mathematica 12. Then, we have shown the validity of the obtained solutions.

***Keywords:*** Generalized Kudryashov method, perturbed Boussinesq equation, soliton solutions

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# AN EXPANSION FORMULA FOR A STURM-LIOUVILLE EIGENVALUE PROBLEM

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**Abstract**

In the present paper, we investigate on the half line $[0,\infty)$ the differential equation in the form

$$-y''+q(x)y=\lambda^2 y, \tag{1}$$

with the boundary condition

$$y'(0)+\left(\alpha_0+\alpha_1\lambda+\alpha_2\lambda^2\right)y(0)=0, \tag{2}$$

where $q(x)$ is a real valued function, $\lambda$ spectral parameter, $\alpha_0,\alpha_1,\alpha_2$ are real numbers.

On the half line $[0,\infty)$, the spectral analysis for the equation (1) with the boundary condition nonlinear dependence on the spectral parameter is studied in [1,2]. Expansion formulas according to the eigenfunctions for differential operators in infinite interval are investigated in [3,4]. The aim of this study is to obtain the expansion formulas of the problem (1)-(2) in terms of scattering data.

***Keywords:*** Expansion formula, Sturm-Liouville operator, resolvent operator.

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# On the Josephson Nonlinear Left-handed Transmission Line

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**Abstract**

In this work, we focus on the deeper properties of the josephson nonlinear left-handed transmission line equation in a detailed manner. Several analytical facts to the governing model are reported.

***Keywords***: Josephson nonlinear left-handed transmission line equation; analitycal method, Complex function solutions.

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# NOVEL RESULTS ON STABILITY OF NONLINEAR FRACTIONAL-ORDER NEUTRAL SYSTEMS WITH TIME VARYING DELAY

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**Abstract**

In this study, some novel results for stability of Riemann Liouville nonlinear fractional-order neutral systems are obtained. Lyapunov functional method and linear matrix inequality technique were used to obtain these results. By constructing a suitable Lyapunov functional associated with fractional integral and derivative terms, several sufficient conditions for asymptotically stability of the equilibrium point are constructed. Illustrative examples are given to show the effectiveness of the proposed theoretical results and MATLAB-Simulink is applied to show the behaviors of the paths of solutions of the considered system for a particular case. Consequently, the obtained results extend and improve some related ones in the literature.

***Keywords:*** Asymptotically stability, Riemann Liouville nonlinear fractional-order neutral systems, Lyapunov functional, Linear matrix inequality.

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# UNIFORM CONVERGENCE RESULTS FOR SINGULARLY PERTURBED FREDHOLM INTEGRO-DIFFERENTIAL EQUATION

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**Abstract**

The study deals with an initial-value problem for a singularly perturbed nonlinear Fredholm integro-differential equation. Parameter explicit theoretical bounds on the continuous solution and its derivative are derived. Parameter uniform error estimates for the approximate solution are established. Numerical results are given to illustrate the parameter-uniform convergence of the numerical approximations.

***Keywords:*** Singular perturbation; Fredholm integro-differential equation; Uniform convergence

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# FINITE DIFFERENCE SCHEMES ON A SHISHKIN TYPE MESH FOR SINGULARLY PERTURBED PROBLEM WITH INTAGRAL BOUNDARY CONDITION

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**Abstract**

In this paper, we deals with a singularly perturbed boundary value problem with integral boundary condition. We propose a uniform convergence numerical method for solving singular perturbed problem with integral boundary condition. The behavior of the exact solution and it's derivative is analyzed for this problem. A numerical method based on a finite difference scheme on a piecewise uniform mesh is constructed. We prove that difference scheme is first order convergent in the discrete maximum norm with respect to singular perturbation parameter. Numerical results agreement with these theoretical results and indicate that the estimates are sharp.

***Keywords:*** singular perturbation, integral boundary condition, finite difference method, uniformly convergence

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# MODELLING OF HEPATITIS-B DISEASE HAVING VERTICAL TRANSMISSION

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**Abstract**

Mathematical modelling is used extensively in natural sciences such as physics, biology, earth science, meteorology, and engineering disciplines such as computer science, artificial intelligence, as well as in social sciences such as economics, psychology, sociology, political science. In addition, mathematical models are mostly used by physicists, engineers, statisticians, operations research analysts and economists to describe and analyse the nonlinear phenomena. In this study, we propose a new illustrative and effective model to point out the behaviours of Hepatitis-B virus (HBV) having vertical transmission which still takes place in the first orders in the list of deadly diseases in the world. Not only we consider the mathematical modelling, equilibria, their stabilities, and existence-uniqueness analysis of the model, but also, we make numerical simulations by using the Adams-Bashforth numerical scheme that is commonly preferred by analysts to solve and simulate both fractional and integer order nonlinear systems. The paper addresses the existence and uniqueness of its solutions via fixed point theory, while the unique non-negative solution remains bounded within the biologically feasible region. The stability analysis of the mentioned model is considered, and the biological relevance of the equilibria is also performed in the paper.

***Keywords:*** Hepatitis-B disease; vertical transmission; fractional order modelling; Caputo fractional derivative; stability analysis; existence-uniqueness; numerical scheme

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3. Mehmet Yavuz, Ndolane Sene, Stability analysis and numerical computation of the fractional predator–prey model with the harvesting rate, Fractal and Fractional, Vol:4, No:3, 35, 2020.

# EXACT AND NUMERICAL SOLUTIONS TO THE BURGERS' AND COUPLED BURGERS' EQUATION

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**Abstract**

In this work, one dimensional Burgers' equation and coupled Burgers' equation are solved via Homotopy perturbation method (HPM). Solutions two and three-dimensional graphics and tables of some obtained results are constructed with the help of the computational program in the Wolfram Mathematica. All the solutions found in this study validate the efficiency of the method. According to the results, we have found out that our gained solutions convergence very speedily to the analytical solutions.

In conclusion, we can say that the present method can also be applied for the solutions of a wide range of nonlinear problems.

***Keywords:*** One dimensional Burgers' equation, Coupled Burgers' equation, Homotopy perturbation method, Embedding parameter.

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# ANALYSIS OF SOME MODELS WITH DIFFERENT KERNELS

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**Abstract**

In this work, we investigate some interesting models. We present the solutions of the models by some integral transforms. We demonstrate the simulations by some figures to prove the efficiency of the proposed transforms. We use four different kernels on the models. We investigate the models with the Caputo derivative, Caputo-Fabrizio derivative, Atangana-Baleanu derivative and constant proportional Caputo derivative.

***Keywords:*** Integral transforms, solutions, simulations, fractional operators.

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# Solving Damped Duffing Equation Using Artificial Neural Networks

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**Abstract**

A simple vibrating system can be modeled using linear spring, damper, and mass elements. Although the linear approach offers a simple solution for some special cases, most systems in nature behave nonlinearly when vibrating. Although non-linear differential equations give realistic results for systems while vibrating, the analytical solutions of these equations are almost non-existent. Duffing equation is one of the differential equations used to model nonlinear vibrating systems. In this study, artificial neural networks are used as an alternative to numerical and analytical methods for the solution of the duffing equation. First, the analytical solution of the duffing equation for damped, non-forced non-linear vibrating systems has been made by using Jacobi elliptic functions in certain initial conditions and different nonlinearity cases. The data obtained were used to train the artificial neural network. Scripts were written using Python programming language to analyze the equation analytically and interpret the data from the neural network. The artificial neural network has been created in MATLAB environment. It was observed that the analytical solution values of different nonlinearity for the initial conditions and the data obtained from artificial neural networks for the steady-state conditions largely overlap.

***Keywords:*** Duffing Equation , Artificial Neural Network , Nonlinearity , Jacobi Elliptic Functions

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# A FINITE DIFFERENCE ANALYSIS OF BOUNDARY VALUE PROBLEMS FOR SINGULARLY PERTURBED EQUATIONS ON BAKHVALOV MESH

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**Abstract**

In this paper we present a special uniform finite difference method on an B-mesh (Bakhvalov type mesh) to solve the boundary value problem for singularly perturbed second order differential equations. The solution of foresold problem, exhibits the boundary layer on the left and right-hand side of the domain due to the presence of singular perturbation parameter $\varepsilon$. For this problem finite difference scheme on a special non-uniform mesh, whose solution converges point-wise independently of the $\varepsilon$ small parameter is constructed and analyzed. The stability and convergence analysis of the method are investigated. The scheme is uniformly convergent, i.e., their convergence is independent of the small perturbation parameter. An error analysis on the scheme shows that the method is of second order convergent in the discrete maximum norm independent of the perturbation parameter, i.e., the scheme are uniformly convergent. Several numerical examples are also given to demonstrate the efficiency of B-mesh to validate the theoretical aspects.

***Keywords:*** Difference scheme; Bakhvalov mesh; Singular perturbation; Uniform convergence.

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# SOME TRIGONOMETRIC APPROXIMATION THEOREMS IN WEIGHTED ORLICZ SPACES

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**Abstract**

In this study, the error of approximation of trigonometric polynomials by means of some special summability methods are investigated in Orlicz spaces with weights satisfying Muckenhoupt conditions. Specially, we proved the approximation theorems in weighted Orlicz spaces using different two matrix methods based on Nörlund and Riesz methods.

Our main results are:

**Theorem 1.** Let $f \in Lip\big(\alpha, L_\Phi(\mathbb{T},\omega)\big), 0<\alpha<1, \omega \in A_{\frac{1}{\alpha_\Phi}}(\mathbb{T}) \cap A_{\frac{1}{\beta_\Phi}}(\mathbb{T})$ and $A=(a_{n,k})$ be a infinite lower triangular matrix with nonnegative entries and row sums 1. If one of the following conditions

$(i)\ \{a_{n,k}\} \in AMIMS,$

$(ii)\ \{a_{n,k}\} \in AMDMS \text{ and } (n+1)(a_{n,n}) = O(1),$

holds, then

$$\|f - T_n(f)\|_{\Phi,\omega} = O(n^{-\alpha}).$$

**Remark.** When $a_{n,n-k} = \frac{p_{n,n-k}}{P_k}$, the method $T_n$ turns into Nörlund method given as

$$N_n(f;x) := \frac{1}{P_n}\sum_{k=0}^{n} p_{n,n-k} S_k(f;x).$$

Here

$$P_n = p_0 + p_1 + p_2 + \ldots + p_n \neq 0 \quad (n \geq 0) \text{ and } p_{-1} = P_{-1} = 0.$$

**Corollary 1.** $f \in Lip\big(\alpha, L_\Phi(\mathbb{T},\omega)\big), 0<\alpha<1, \omega \in A_{\frac{1}{\alpha_\Phi}}(\mathbb{T}) \cap A_{\frac{1}{\beta_\Phi}}(\mathbb{T})$ and $(p_n)$ be a positive sequence. If one of the following conditions

$(i)(p_n) \in AMIMS,$

$(i)\ (p_n) \in AMDMS \text{ and } (n+1)p_n = O(P_n),$

holds, then

$$\|f - N_n(f)\|_{\Phi,\omega} = O(n^{-\alpha}).$$

Similar results can be obtained using matrix transformation based on Riesz method.

***Keywords:*** Trigonometric approximation, matrix method, error of approximation, weighted Orlicz Spaces.

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# COMPARISON OF MONTE CARLO SIMULATION AND NUMERICAL METHODS FOR STOCHASTIC DIFFERENTIAL EQUATIONS

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**Abstract**

This paper comparatively investigates Monte Carlo simulation and some numerical methods to solve stochastic differential equations (SDEs). At the present paper, SDEs with initial conditions are considered. We approximate to numerical solution using Monte Carlo simulation and numerical methods. To show the effectiveness of the Monte Carlo and numerical methods, approximation solutions are compared with exact solution. And finally the results of Monte Carlo and numerical methods are supported with graphs and error tables and discussed about which method is useful and superior and in which cases these methods can be used.

***Keywords:*** Monte Carlo Simulation ; Stochastic differential equations; Numerical Methods.

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# Highly Ultra-Broadband QD-SOA Exploiting Superimposition of Quantum Dots

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## Abstract

In this paper, for the first time, a highly ultra-broadband QD-SOA based on superimposed Quantum Dots has been proposed to accomplish a broader optical gain spectra range of almost 3.5μm from blue to Mid-infrared, considering solution-processed nanotechnology as a simple and cost-effective fabrication method. The realization of the ultra-broadband optical gain can be accomplished by the superimposition of the various size-distributed QD groups made of different materials in a way that the broader energy span covered as a consequence of the variation of the bandgap energy for each QD groups. To this end, different QD groups made of Bismuth Tellurium Sulfide ($Bi_2Te_{3-x}S_x$) and Cadmium Tellurium Sulfide ($CdTe_{1-x}S_x$) in the ZnS shell implemented in the active region of QD-SOA have been superimposed, in which the radii of each QD groups can be easily distributed due to incorporating solution-processed method. The performance of the proposed QD-SOA has been modeled based on the developed rate equation framework by assuming inhomogeneous broadening of energy levels as a result of the size distribution of QDs and the superimposition of various QD groups. Furthermore, the bandwidth and, the spectral range, and the flatness of the optical gain in the QD-SOA can be managed by the number of QD groups, the percentage of Tellurium and Sulfur in the $Bi_2Te_{3-x}S_x$ and $CdTe_{1-x}S_x$ alloys, and the size distribution of each QD groups.

***Keywords:*** Semiconductor Optical amplifier, QDs, Superimposed QDs, Broadband SOA

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# A proposal for Wide range Upconversion Process using Optical Force

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## Abstract

Optomechanical wavelength up-conversion based on optical force and core-shell scattering effects are used to control light coupling between two waveguides. This system consists of two parallel optical waveguides with 20 $\mu m$ lengths suspended on a silica substrate embedded with $Ag/Si/SiO_2$ core-shell nanoparticles. By Mid-IR plane wave illumination with different intensities and different wavelengths on nanoparticles, scattering would increase and result in an improvement in attractive gradient optical force exerted on waveguides. Via bending waveguides toward each other, NIR light propagating in the first waveguide would couple to another. PDMS as a polymer is used to reduce the required power for bending waveguides. Results reveal that when waveguides' gap equilibrium is 0.5 $\mu m$ and wavelengths of control and probe lights are 3.4 $\mu m$ and 1310 nm respectively, about 16.5 $mW/\mu m^2$ power is needed to bend waveguides for total coupling of light between waveguides.

***Keywords:*** Optomechanics, Optical force, Upconversion, Scattering, QDs.

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# High-Efficiency Integrated MIR Band to Visible Band Upconversion Optoelectronic System

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## Abstract

In this paper, we introduce and investigate an ultra-high efficiency up-converter to convert three mid-infrared wavelengths bands to three visible wavelengths bands. Three thin layers of the photoconduction nanocrystals of PbSe, CdSe, and CdTe doped were put on the Base of the NPN bipolar junction transistor as photodetectors and three doped phosphorescence organic light-emitting diodes in different colors are placed on the Collector contacts. In this article, we perform the frequency analysis of colloidal quantum dot infrared photodetectors and up-conversion process implementation by the proposed structure. According to our surveys, this is the first study about the frequency response of the up-conversion device. First, rate equations of three-wavelength quantum dot photodetectors are presented to find the photocurrents of the device. Then, we analyze the reaction of the up-conversion device to different frequencies of input light signals. The quantum dot photodetectors were designed to detect three different central wavelengths of 3, 4.3, and 5.3μm and also blue, green, and red organic light-emitting diodes operated at central wavelengths of 450, 550, and 620nm, respectively. In this investigation, narrow-band infrared light was converted into broadband visible light considering conventional light-emitting diodes. The mid-infrared input optical signal with a bandwidth of 0.004, 0.0006, and 0.004THz was converted to visible optical bandwidth of 0.425, 0.52, and 0.37THz, respectively. It should mention that considering specific designs for LEDs and QDs it is possible to convert the desired band in MIR to the desired visible band. The results showed that the output quantum efficiency of the proposed structure for 3, 4.3, and 5.3μm was 100, 500, and 200, respectively. Also, the enhancement of transistor current gain (β) made a rise in the quantum efficiency of the designed device.

***Keywords:*** Band to band upconversion, QDs, LEDs, Optoelectronic integrated system

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# Broadband Terahertz Absorber using superimposed Graphene Quantum Dots

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## Abstract

In this study, we propose and investigate numerically a broadband THz metamaterial-based absorber, which is composed of superimposed Graphene Quantum Dots. Based on this idea, a new way to engineering the absorption band is introduced. We will show that using the proposed idea, it is possible to design a THz absorber with a given bandwidth. To show the capability of the idea, we consider a three-layer structure, and the top layer is superimposed graphene disks. The middle layer is a lossless dielectric thin layer and eventually gold is placed in the bottom layer. Simulation results reveal a broadband absorption in the range of (5.86THz to 7.57THz), (5.89THz to 7.56THz), and (5.89THz to 7.58THz) while absorption values respectively are above 89%, 88.49%, and 88.32%. The dielectric material is $Si_3N_4$ in the proposed structure. Also, the broadband absorption range is 7.47THz to 9.87THz with an absorption value above 80% while the dielectric material is $SiO_2$.

***Keywords:*** Broadband absorber, graphene, metamaterial

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# A Proposal for Sub diffraction Optical Imaging

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## Abstract

In this work, a fast and new sub-diffraction limit, fluorescent marker-free and nonlinear optical imaging method have been proposed to reconstruct the refractive index variation profile. To do this, a one-dimensional approach is developed in this article. To this end, the superposition of optical signals with controlled phase information for the reconstruction of the refractive index of a point on target is used. By tuning the phase information, it is possible to reconstruct the refractive index for the whole media. After the phase modulator (PM) in the proposed system, a harmonic generator black box ($5^{th}$ harmonic in this work) is used. Considering this block it is possible to enhance the resolution of the proposed imaging system. The proposed method can reconstruct the refractive index variation profile below the subdiffraction limit. After a harmonic generator block, a passive optical combiner is used and the result is converted to an electric signal through the photodetector. The electric signal is used to reconstruct the refractive index variation of the unknown media. Finally, the proposed method is evaluated numerically and dependency of the resolution of the imaging system upon system parameters is analyzed too.

***Keywords:*** Rayleigh diffraction limit, subdiffraction limit imaging, Nonlinear electronic system, Index of refraction tomography

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# Selective Band Amplification in the Ultra Broad Band Reflective Semiconductor Optical amplifiers

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**Abstract**

In this paper, a novel approach has been proposed for having an ultra-broadband quantum dot reflective semiconductor optical amplifier (QD-RSOA) using superimposed quantum dots with switching and band selection capability in the supported band. Furthermore, about 1 μm optical bandwidth has been covered (O, E, S, C, and L bands), which is the desired region in most optical communication applications. Three optical windows have been selected for having an optimized amplification (1.55 μm, 1.5 μm, 1.31 μm). Also, they can either be amplified simultaneously or one at a time, which guarantees independent modulation. This is a remarkable property in fast data transmission. Besides, this amplifier has been devised by solution process nanotechnology, which guarantees its synthesizing feasibility with low costs. Finally, by introducing this amplifier one step is taken to the development of fast WDM-PON's.

***Keywords:*** Reflective Semiconductor Optical amplifier, QDs, Superimposed QDs, Broadband RSOA

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# Analyzing the Wave Solutions of the Mathematical Model with the Modified Exponential Function Method

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**Abstract**

In this study, traveling wave solutions related to the (3+1)-dimensional variable coefficients Date-Jimbo-Kashiwara-Miwa (VC-DJKM) equation given as a mathematical model are obtained by using the modified exponential function method. Solution functions found as a result of all these calculations include hyperbolic, trigonometric, and rational functions. Two-dimensional, three-dimensional, contour graph and density graphs obtained by determining appropriate solution functions represent the characteristic feature of solution functions.

**Keywords:** The traveling wave solutions; the Dullin-Gottwold-Holm equation; the modified exponential function method.

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# A FRACTIONAL-ORDER MODEL FOR THE DYNAMICS OF HIV INFECTION WITH CAPUTO DERIVATIVE

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**Abstract**

The objective of this research study is to provide a detailed exposition of the HIV infection model by means of the Caputo fractional operator. In the world, HIV/AIDS (Human Immunodeficiency Virus/Acquired Immune Deficiency Syndrome) is one of the most crucial public health problems owing to its transmission from person to person, its spread to large masses, and severe health and socioeconomic consequences. The fact that the world is getting smaller and integrated with the developing technology increases the spread of HIV similar to other infectious diseases. Hence, it is undoubtedly helpful to thoroughly examine the HIV infection, taking advantage of the non-local fractional operator. In addition to the obtained theoretical results and some basic properties of the fractional-type disease model under investigation, we evaluate the dynamic behavior of the model with the help of a sufficient numerical scheme under the Caputo operator. On the other hand, it is worth mentioning that we take into account the dimensional consistency during the fractionalization process of the traditional version of the suggested disease model.

***Keywords:*** Fractional operators, HIV infection, Fractional Euler method, Fractional modeling, Caputo derivative.

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# ANALYSIS OF FRACTIONAL LANA FEVER INFECTION MODEL INCLUDING SINGULAR KERNEL

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**Abstract**

The core of this study is to provide a comprehensive analysis of the lana fever infectious ailment by means of the Caputo fractional operator. The main purpose of utilizing non-integer order derivative is to comprehend the dynamics of the lana fever disease transmission in detail. For this aim, we present such crucial analyzes as the existence and uniqueness of the solution of the proposed fractional disease model and finding the most sensitive parameters by employing forward sensitivity index in relation to reproduction number $R_0$. Moreover, we show that the disease-free equilibrium is locally asymptotically stable when $R_0 < 1$ and the endemic equilibrium of the model is globally asymptotically stable when $R_0 > 1$. On the other hand, numerical simulations are performed in order to observe the behavior of the solutions curves of the model under investigation for different values of non-integer order $\alpha$.

***Keywords:*** Fractional derivatives, Lana fever, Fractional modeling, Mathematical biology.

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# A Novel Analytical Method to the Equation System of Ion-acoustic and Langmuir Waves in Plasmas Physics

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**Abstract**

In this research, we apply a newly developed analytical scheme which is the rational sine-Gordon expansion method to construct novel solutions of a governing model for the ion-acoustic and Langmuir waves. All the solutions achieved have been analyzed graphically to reveal their physical properties. We concluded that the proposed method is an efficient method that gives more general analytical solutions to powerfully nonlinear partial differential models.

***Keywords:*** The rational sine-Gordon expansion method; ion-acoustic wave; Langmuir wave; plasma physics; solitary wave solutions

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# NUMERICAL SOLUTION FOR NONLINEAR SINGULARLY PERTURBED PROBLEM WITH TWO INTEGRAL BOUNDARY CONDITIONS

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**Abstract**

In this study, singularly perturbed nonlinear boundary value problem with integral boundary conditions is considered. The problem is numerically solved by finite difference method on Bakhvalov-Shishkin type mesh. The convergence analysis of the proposed method is derived. The difference method is uniformly convergent with respect to perturbation parameter in discrete maximum norm. Moreover, numerical experiment is presented to show the performance of the proposed method.

***Keywords:*** Singular perturbation equation, finite difference method, non-uniform mesh, uniform convergence, integral boundary conditions.

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3. N.S. Bakhvalov, On optimization of methods for solving boundary value problems in the presence of a boundary layer, Zhurnal Vychislitel'noi Matematikii Matematicheskoi Fiziki, Vol: 9, 841-859, 1969.

# NUMERICAL SOLUTION FOR NONLINEAR SINGULARLY PERTURBED PROBLEM WITH TWO MULTIPOINT BOUNDARY CONDITIONS

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**Abstract**

In this study, singularly perturbed nonlinear boundary value problem with multi-point boundary conditions is considered. The problem is numerically solved by finite difference method on Shishkin type mesh. The convergence analysis of the proposed method is derived. The difference method is uniformly convergent with respect to perturbation parameter in discrete maximum norm. Moreover, numerical experiment is presented to show the performance of the proposed method.

***Keywords:*** Singular perturbation equation, finite difference method, non-uniform mesh, uniform convergence, multi-point boundary conditions.

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3. N.S. Bakhvalov, On optimization of methods for solving boundary value problems in the presence of a boundary layer, Zhurnal Vychislitel'noi Matematikii Matematicheskoi Fiziki, Vol: 9, 841-859, 1969.

# DIFFICULTIES IN DISTANCE EDUCATION FROM THE PERSPECTIVE OF MATHS EDUCATION AND MATHEMATICS DEPARTMENTS FACULTY MEMBERS

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**Abstract**

In order to realize the learning process efficiently and effectively, it is necessary to benefit from the use of technology in the education and training process. It can be said that the use of technology and learning techniques, which have positive effects on instructors and students, are very beneficial in terms of both gaining new experiences in the education world, supporting the person cognitively, and gaining skills to the individual. On the other hand, it is thought that technological facilities used in distance education during the pandemic process have some deficiencies besides the contributions that students and instructors encounter. In this context, the research will be carried out with faculty members of the Faculty of Education, Elementary Mathematics Education and the Faculty of Arts and Sciences, Mathematics departments of a state university in the Eastern Anatolia Region, in Turkey. In this way, it is aimed to examine the competencies and deficiencies of the digital education systems used with distance education for mathematics teaching. It is great important to get the opinions of academics regarding this situation in terms of making the necessary improvements in the future. As a matter of fact, mathematics teachers stated that distance education is not effective and efficient on mathematics education (Kilit & Güner, 2021). In this respect, it is thought that determining the views of academicians on the subject in higher education will contribute to the field. This research is a descriptive study using qualitative method. The analysis results of the findings obtained from the research will be presented in the conference.

***Keywords:*** Mathematics Field Course, Digital Education, Distance Education, Faculty Staff.

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# Speaker Identification using Long Short-Term Memory and GMM

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**Abstract**

Audio knowledge plays an important role in the increasing digital content available today. It requires methodologies that analyze such content. Speaker Identification is important research areas against Audio Signals. Speaker Identification is known as the task of recognizing the person speaking without speaking. Speaker Identification, transaction verification, access control, voice dialing, web services, etc. Including is likely. Speech Recognition, Speech-to-Text Conversion and vice versa, etc. The Mel Frequency Cepstral Coefficient (MFCC) is considered to be a factor in performing Speaker Identification. Trainig is the deep learning process of the model training phase for a speaker identification project; Artificial Neural Networks (ANN). In this paper the long Short Term Memory Networks (LSTM) architecture, which is a deep learning model for traning, is compared with the GMM model using the TIMIT dataset. All obtained results have been analyzed and their accuracy and error rates are presented.

***Keywords:*** Speaker Identification, Audio Signals, LSTM, TIMIT, GMM

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# Fractional vector-borne disease model with lifelong immunity under Caputo operator

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**Abstract**

In the current study, we analyze and investigate the vector-borne disease model with lifelong immunity by means of the Caputo fractional differential operator. Some crucial theoretical and numerical results with various graphs are presented by taking advantage of different values of non-integer order $\alpha$. On the other hand, the existence and uniqueness of the solution of the fractional vector-borne disease model are shown by employing the fixed-point theorem. Also, for the disease model under consideration, more detailed results are obtained thanks to the fractional-order derivative as can be seen from the solution curves in the graphs.

***Keywords:*** Caputo fractional derivative, Fractional modeling, Mathematical biology.

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# ANALYSIS OF FRACTIONAL DIFFERENTIAL EQUATIONS WITH DIFFERENT KERNELS

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**Abstract**

In the area of fractional calculus, various types of differential operators were introduced. In this paper, we analyze the fractional differential equations with different kernels. We apply very effective numerical techniques to get the solutions of the problems. We show some numerical results to prove the accuracy of the proposed methods.

***Keywords:*** Fractional differential equations, numerical methods, simulations.

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# Dynamical behaviors of a conformable fractional order logistic model

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In this study, we have considered a conformable fractional order logistic model with piecewise constant arguments. First of all, the piecewise constant approximation is used to obtain the discretize version of the model. Then, we obtain necessary and sufficient stability conditions by using the Schur Cohn criterion. Moreover, bifurcation analysis show that Neimark-Sacker bifurcation occurs according to the change of parameter $r$ in the system. Finally, we also obtain bifurcation diagrams, phase portraits and Lyapunov exponents by the numerical simulations.

**MSC 2010:** 26A33, 34A08, 39A28, 39A30, 92B05
**Keywords:** Conformable fractional derivative, Piecewise constant arguments, Stability, Neimark-Sacker bifurcation

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# Approximate-analytical method for solving the nonlinear fractional Ebola Virus mathematical model

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**Abstract**

The presented work aims to design a novel nonlinear mathematical model of Caputo fractional order to describe the new Ebola virus. This study aims to present different scenarios where we apply an effective analytical Laplace Adomian decomposition method (LADM) for simulating such a model. All obtained solutions have been analyzed and compared graphically to validate the efficiency and applicability of all results.

***Keywords:*** Ebola virus model, Caputo fractional derivative, Laplace Adomian decomposition method (LADM).

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# On New Integral Inequalities via Atangana-Baleanu Fractional Integral Operators

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**Abstract**

The main purpose of this study is to bring together the field of inequalities with Atangana-Baleanu fractional integral operators, which are the focus of attention among fractional integral operators with their features and frequency of use. For this purpose, after introducing some basic concepts, a new variant of Hadamard's inequality is obtained for s-convex functions in the second sense. Then, an integral equation, which is important for the main findings, has been proved. With the help of this integral equation that includes Atangana-Baleanu fractional integral operators, many Hadamard type integral inequalities are obtained for the functions whose absolute values of the second derivatives are s-convex and s-concave. Some classical inequalities and hypothesis conditions such as Hölder's inequality and Jensen's inequality were taken into account in the proof of the findings.

***Keywords:*** Hermite-Hadamard inequality, Hölder inequality, Atangana-Baleanu integral operators, Normalization function, Euler Gamma function, Euler Beta function

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# TWO NUMERICAL METHOD FOR FRACTIONAL ORDER PSEUDO-PARABOLIC DIFFERENTIAL EQUATIONS

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**Abstract**

In this study, all basic definitions are given for Fractional Caputo differential equations. Fractional pseudo-parabolic equation with initial-boundary conditions is investigated. Modified Double Laplace Decomposition Method is used for the exact solution of this equation. Explicit finite difference is constructed for this differential equation. Stability estimates are proved for these difference schemes. Error analysis and figures are made by comparing the exact solution and the approximate solutions. Figures showing the physical properties of the exact and approximate solutions are presented. From the error analysis table and figures, it is clearly seen that this applied method is an effective and good method for this equation.

***Keywords:*** Fractional order pseudo-parabolic differential equation, Explicit difference method, modified double Laplace decomposition method, stability, numerical solution.

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# ON THE KUMARASWAMY-GARIMA DISTRIBUTION AND STATISTICAL PROPERTIES

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This article introduces a new distribution called the Kumaraswamy-Garima distribution. The Kumaraswamy distribution is very similar to the Beta distribution but has the key advantage of a closed-form cumulative distribution function. The Garima distribution is proposed as a model for lifetime. Some important statistical properties of the new distribution like cumulative distribution function, hazard rate, failure rate, inverse hazard function, odd function and the cumulative hazard function, r th moment, moment generating function characteristic function, moments, mean and variance, Bonferoni and Lorenz curve**,** order statistics, MLE, mean time between failures (MTBF) has been derived. As a result, the classical properties we have just found are presented in graphics.

**Key Words:** Kumaraswamy Garima Distribution, Odd function, Order Statistics, Moment Generating Function and Characteristic Function

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# Development Processes of a Capacitive Rotary Encoder

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## Abstract

In this study, capacitive encoder development process is presented. Encoders are key elements for various applications that require angular position data. However, there are restrictions on the adaptation of optical and magnetic encoders to different application fields. With capacitive sensing technology, restrictions can be overcome. In order to develop encoders that can be easily adapted to different areas, capacitive encoder technology should be well analyzed and the encoder development process should be handled in details. In most capacitive encoder studies, this process does not seem to work properly. Here, a general capacitive encoder architecture is given considering the compatibility of capacitive encoder mechanics and electronics. Capacitive encoders are classified in terms of their basic components. The mathematical model of the encoder determined according to the classification has been acquired with the modeling method, which is useful for similar types capacitive encoders. Encoder outputs were observed through the mathematical model of the capacitive encoder and angular position measurement was performed. Suggestions have been made to increase and facilitate the capacitive measurement quality.

***Keywords:*** Capacitive Sensing; Capacitive Sensor Development; Capacitive Type Encoders; Demodulation.

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# CASCADE FUNCTION BASED RANDOM NUMBER GENERATOR

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## Abstract

Cascade function is designed by combining two seed maps that resultantly has more parameters, high complexity, randomness, and more unpredictable behavior. In the paper, a cascade fractal function, i.e. cascade-PLMS is proposed by considering the phoenix and lambda fractal functions. The constructed cascade-PLMS exhibits the required fractal features such as fractional dimension, self-similar structure, and covering entire phase space by the data sequence in addition to the chaotic properties. Due to the chaotic behavior, the proposed function is utilized to generate a pseudo-random number sequence in both integer and binary format. This is the result of an extreme scalability feature of a fractal function that can be implemented on a large scale. A sequence generator is designed by performing the linear function operation to the real and imaginary part of a cascade-PLMS, cascade-PLJS separately, and the iteration number at which the cascade- LJS converges to the fixed point. The performance analysis results show that the given method has a large key space, fast key generation speed, high key sensitivity, and strong randomness. Therefore, the scheme can be efficiently used further to design a secure cryptosystem with the ability to withstand various attacks.

***Keywords:*** Cascade phoenix lambda fractal, PRNG, Mandelbrot set, Dynamic behavior, Key security analysis.

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# Numerical solution of Pavlov equation by The Adomian decomposition method and homotopy perturbation method

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## Abstract:

In the present article, two numerical method are successfully applied to study the invariance properties of the $(2+1)$-dimensional Pavlov equation. The Adomian decomposition method and homotopy perturbation method are used to study the suggested equation. The obtained solutions are examined with exact solution.

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# Hyperbolic Padovan and Hyperbolic Pell-Padovan Sequences

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**Abstract**

In this article, we extend Padovan and Pell-Padovan numbers to Hyperbolic Padovan and Hyperbolic Pell-Padovan numbers, respectively. Moreover we obtain Binet-like formulas, generating functions and some identities related with Hyperbolic Padovan and Hyperbolic Pell-Padovan numbers.

***Keywords:*** Padovan numbers, Pell-Padovan numbers, Hyperbolic numbers, Hyperbolic Padovan numbers, Hyperbolic Pell-Padovan numbers.

# NEW BEHAVİOURS OF THE (2+1)-DİMENSİONAL CAMASSA-HOLM-KADOMTSEV-PETVİASHVİLİ AND THE NONLİNEAR ZOOMERON EQUATİONS VİA THE SİNE-GORDON EXPANSİON METHOD

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**Abstract**

This study offer solutions of the (2+1)-dimensional Camassa-Holm-Kadomtsev-Petviashvili and the non-linear Zoomeron equations by the sine-Gordon expansion method which is one of the strong and most popular methods for attain complete solutions the non-linear partial differential equations. By this present method, many exact solitary wave solutions like topological, non- topological and consolidate topological are attained and supported with 3D and 2D graphs by selecting suitable parameters.

***Keywords:*** sine-Gordon expansion method, NLEEs, wave solutions.

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# New solutions to the B-type Kadomtsev-Petviashvili-Boussinesq Equation in (3+1) dimensions

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**Abstract**

In this study were presented $\left(\frac{1}{G'}\right)$-expansion method to the (3+1)-dimensional B-type Kadomtsev-Petviashvili-Boussinesq Equation. We achieved some wave solutions such as new travelling wave solutions. By using computational program, obtained three dimensional surfaces of the results. In addition to end of this manuscript, we submit a conclusion in the comprehensive manner.

***Keywords:*** (3+1)-dimensional B-type Kadomtsev-Petviashvili-Boussinesq equation, $\left(\frac{1}{G'}\right)$-expansion method, travelling wave solutions.

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# AN EXTENSION OF THE INVERSE GAUSSIAN DISTRIBUTION

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**Abstract**

In this study, an extension of the inverse Gaussian distribution is introduced; see Tweedie (1957a, 1957b) for details about the inverse Gaussian distribution. This new distribution is obtained using the philosophy of the $\alpha$-monotone distribution given in Jones (2020). Therefore, it is called an α-monotone inverse Gaussian ($\alpha$IG) distribution. Some characteristic measures, e.g., expected value, variance, skewness, and kurtosis, of the $\alpha$IG distribution are obtained. The parameters of the $\alpha$IG distribution are estimated via the method of moments. A real data set is modeled by using the $\alpha$IG distribution to show its implementation.

***Keywords:*** $\alpha$-monotone distribution, Inverse Gaussian distribution, method of moments

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# NOTES ON ENDOMORPHISM RINGS OVER LEAVITT PATH ALGEBRA

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**Abstract**

If E is a directed graph and K a field, $L_K(E)$ denotes the Leavitt path algebra of E over K. $L_K(E)$ was introduced independently by Abrams and Aranda Pino [1], and by Ara, Moreno and Pardo [4], using different approaches. In this article, we study a current account of the endomorphism rings of Leavitt path algebra, and its relations with various ring theory.

***Keywords:*** Endomorphism rings, Regular rings, Strongly regular rings, Leavitt path algebra.

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# CASCADE FUNCTION BASED RANDOM NUMBER GENERATOR

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## Abstract

Cascade function is designed by combining two seed maps that resultantly has more parameters, high complexity, randomness, and more unpredictable behavior. In the paper, a cascade fractal function, i.e. cascade-PLMS is proposed by considering the phoenix and lambda fractal functions. The constructed cascade-PLMS exhibits the required fractal features such as fractional dimension, self-similar structure, and covering entire phase space by the data sequence in addition to the chaotic properties. Due to the chaotic behavior, the proposed function is utilized to generate a pseudo-random number sequence in both integer and binary format. This is the result of an extreme scalability feature of a fractal function that can be implemented on a large scale. A sequence generator is designed by performing the linear function operation to the real and imaginary part of a cascade-PLMS, cascade-PLJS separately, and the iteration number at which the cascade- LJS converges to the fixed point. The performance analysis results show that the given method has a large key space, fast key generation speed, high key sensitivity, and strong randomness. Therefore, the scheme can be efficiently used further to design a secure cryptosystem with the ability to withstand various attacks.

***Keywords:*** Cascade phoenix lambda fractal, PRNG, Mandelbrot set, Dynamic behavior, Key security analysis.

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# Differential Operators in the Homogeneous Space of $SU(3)$ Group

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**Abstract**

The metric matrix and Laplace-Beltrami operator is expressed according to a parametric expression of the homogeneous space of $SU(3)$ group. Then, the solution of the eigenvalue-eigenfunction problem obtained by the operator was given. The wave functions of the physical system derived from the solution were obtained and, the potential function and the wave function of the system were drawn. Then, the other differential operators of the $SU(3)$ group, $I^2, I_3, Y$ operators were expressed. Similarly we were gave the solution of the eigenvalue-eigenfunction problem induced by these operators.

***Keywords:*** SU(3) Lie group, homogeneous space, differential operator.

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# Optical soliton solutions of Fokas-Lenells equation via $\left(m+\frac{1}{G'}\right)$-expansion method

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***Abstract:***

In this research paper, we investigate some novel soliton solutions to the perturbed Fokas-Lenells equation by using the $\left(m+\frac{1}{G'}\right)$-expansion method. Some new solutions are obtained and they are plotted in two and three dimensions. This technique appears as a suitable, applicable and efficient method to search for the exact solutions of nonlinear partial differential equations in a wide range. All gained optical soliton solutions are substituted into the Fokas-Lenells equation, and they verify it, the constraint conditions are also given.

***Keywords:*** Optical soliton solutions; $\left(m+\frac{1}{G'}\right)$-expansion method; Fokas-Lenells equation

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# ANALYSIS OF THE SPREAD OF HOOKWORM INFECTION WITH CAPUTO-FABRIZIO FRACTIONAL DERIVATIVE

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***ABSTRACT***

This research study provides a mathematical analysis for the spread of Hookworm infection model. Firstly, the proposed disease model is extended by means of the Caputo-Fabrizio fractional derivative. Then, existence and uniqueness of the solution is presented for the fractional-type Hookworm infection model with the help of the fixed-point theorem. Theoretical results of the model under consideration show the advantages of the fractional differential operators.

**Keywords:** Caputo-Fabrizio fractional derivative, modeling, existence and uniqueness

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# TRAJECTORIES GENERATED BY SPECIAL SMARANDACHE CURVES ACCORDING TO POSITIONAL ADAPTED FRAME

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**Abstract**

In differential geometry, the theory of curves has an important place. The concept of moving frames defined on curves is an important part of this theory. Recently, Özen and Tosun have introduced a new moving frame for the trajectories with non-vanishing angular momentum in 3-dimensional Euclidean space (J. Math. Sci. Model. 4(1), 2021). This frame is denoted by {**T**, **M**, **Y**} and called as positional adapted frame. In the present study, we investigate the special trajectories generated by **TM**, **TY** and **MY**-Smarandache curves according to positional adapted frame in $E^3$ and we calculate the Serret-Frenet apparatus of these trajectories. Later, we consider a specific curve and obtain the parametric equations of the aforesaid special trajectories for this curve. Finally, we give the graphics of these obtained special trajectories which were drawn with the mathematica program. The results obtained here are new contributions to the field. We expect that these results will be useful in some specific applications of differential geometry and particle kinematics in the future.

***Keywords:*** Angular momentum, Kinematics of a particle, Moving frame, Smarandache curves

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# ON THE INFINITY POINT OF THE ELLIPTIC CURVES

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**Abstract**

An elliptic curve is a smooth, projective, algebraic curve, together with an element O called "the point at infinity". Elliptic curves have been studied in algebraic geometry and number theory by mathematicians for a very long time. Elliptic Curve (EC) systems having been playing important role in public-key cryptography and its traditionally implemented using the multiplicative group of a finite field. The first instances of elliptic curves occur in the work of Fermat. Firstly, In 1984 H. Lenstra showed how elliptic curves to factorization of integers and then Elliptic Curve (EC) systems as applied and to cryptography. Neal Koblitz and Victor Miller proposed how elliptic curves could be implement public key protocols independently in 1985. In this study, we give properties of elliptic curves in Cryptography and investigate it's point at infinity algebraicaly.

***Keywords:*** Elliptic curves, Elliptic curve cryptography, Projective coordinates, point at infinity

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# THE SOLUTION OF THE SECOND PART OF THE 16TH HILBERT PROBLEM FOR SOME FAMILIES OF DISCONTINUOUS PIECEWISE DIFFERENTIAL SYSTEMS

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**Abstract**

We provide the exact maximum number of crossing limit cycles of six families of discontinuous piecewise differential systems formed by two differ ential systems separated by a straight line, when these differential systems are linear centers or three families of cubic inilpotent centers. In particular, by using the first integrals of these systems we have solved the extension of the second part of the 16th Hilbert problem to these classes of discontinuous piecewise differential systems.

***Keywords:*** Discontinuous piecewise differential systems, nilpotent cubic systems, linear differential centers, limit cycles.

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